ECE 6345

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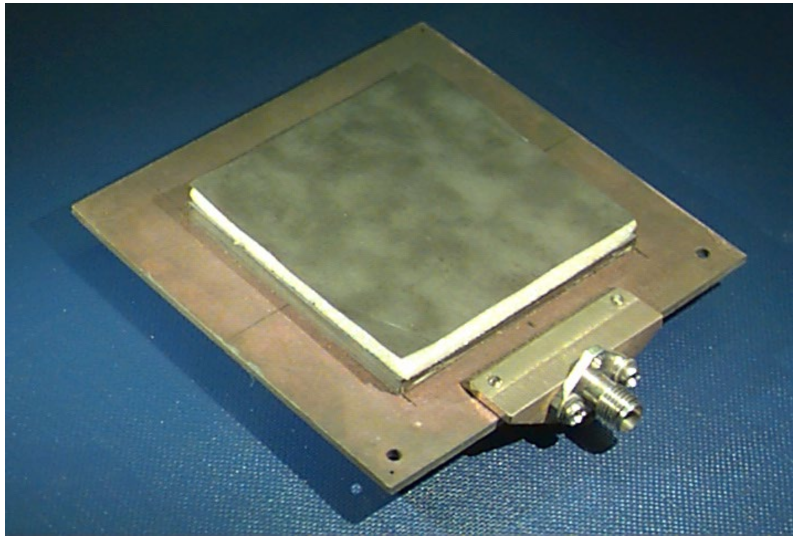


Notes 15

Overview

In this set of notes we derive a CAD formula for the space-wave Q factor (Q_{sp}) of the rectangular patch, using the previously-derived CAD formula for the space-wave radiated power.

Calculation of Q_{sp}

Recall:

$$Q_{\text{sp}} = \omega_0 \left(\frac{U_s}{P_{\text{sp}}} \right) = \omega_0 \left(\frac{2U_H}{P_{\text{sp}}} \right)$$

We have, from Notes 12,

$$P_{\text{sp}} = p \, P_{\text{sp}}^{\text{dip}}$$

$$\approx p \left(\frac{2}{\pi} WL \right)^2 (k_0 h)^2 \, k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1$$

$$c_1 \equiv 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

Where it is assumed that $J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L} \right)$ (The origin is at the center of the patch.)

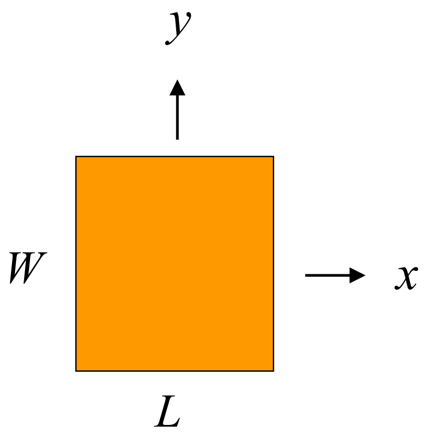
Calculation of Q_{sp} (cont.)

To calculate the stored magnetic energy, use $\underline{H} = \underline{\hat{y}} \cos\left(\frac{\pi x}{L}\right)$ $\quad \left(\underline{J}_s = \underline{\hat{n}} \times \underline{H} = -\underline{\hat{z}} \times \underline{H}\right)$

$$J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right)$$

The stored magnetic energy is then:

$$
\begin{aligned}
U_H &= \frac{1}{4} \int_V \mu_0 \mu_r H_y^2 \, dV \\
&= \frac{1}{4} h \int_S \mu_0 \mu_r H_y^2 \, dS \\
&= \frac{1}{4} h W \mu_0 \mu_r \int_{-L/2}^{L/2} \cos^2\left(\frac{\pi x}{L}\right) dx \\
&= \frac{1}{4} h W \mu_0 \mu_r \left(\frac{L}{2}\right) \\
&= \frac{1}{8} \left(hWL\right) \mu_0 \mu_r
\end{aligned}
$$



$$U_s = \frac{1}{4} \left(hWL\right) \mu_0 \mu_r$$

Calculation of Q_{sp} (cont.)

Hence, we have:

$$Q_{sp} = \omega_0 \left(\frac{\frac{1}{4} hWL \ \mu_0 \mu_r}{P_{sp}} \right) = \omega_0 \left(\frac{\frac{1}{4} hWL \ \mu_0 \mu_r}{p P_{sp}^{\mathrm{dip}}} \right)$$

Formulas for the p factor and the dipole space-wave power are given on the next slide.

Calculation of Q_{sp} (cont.)

$$P_{sp}^{dip} = (Il)^2 k_0^2 \left(\frac{\eta_0}{32\pi}\right) \int_0^{\pi/2} \left[|F(\theta)|^2 + |G(\theta)|^2 \right] \sin\theta \, d\theta$$ (exact)

$$P_{sp}^{dip} \approx (Il)^2 (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi}\right) \mu_r^2 c_1$$ (approximate from Notes 12)

$$c_1 \equiv 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

$$Il = \frac{2}{\pi} WL$$

$$\left(J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right) \right)$$

$$p = \frac{\int_0^{2\pi}\int_0^{\pi/2} \left[|F(\theta)|^2 \sin^2\phi + |G(\theta)|^2 \cos^2\phi \right] |A(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \left[|F(\theta)|^2 \sin^2\phi + |G(\theta)|^2 \cos^2\phi \right] |A(0,0)|^2 \sin\theta \, d\theta \, d\phi}$$ (exact p from Notes 13)

(See Notes 12 for the F and G functions.)

$$A(\theta,\phi) = \left(\frac{\pi}{2} WL\right) \operatorname{sinc}\left[k_y \frac{W}{2} \right] \left[\frac{\cos\left(k_x \frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(k_x \frac{L}{2}\right)^2} \right]$$

$$p \approx 1 + \frac{a_2}{10}(k_0 W)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)(k_0 W)^4 + c_2\left(\frac{1}{5}\right)(k_0 L)^2 + a_2 c_2 \left(\frac{1}{70}\right)(k_0 W)^2 (k_0 L)^2$$ (approximate p from Notes 13)

$$k_x = k_0 \sin\theta \cos\phi$$
$$k_y = k_0 \sin\theta \sin\phi$$

CAD Formula for Q_{sp}

Approximating, we have:

$$Q_{\text{sp}} \approx \frac{\omega_0 \left(\frac{1}{4} hWL \right) \mu_0 \mu_r}{p \left(\frac{2}{\pi} WL \right)^2 (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1}$$

Simplify, using: $k_1 L = \pi$

or $k_0 L \sqrt{\mu_r \varepsilon_r} = \pi$

so $k_0 = \dfrac{\pi}{L} \dfrac{1}{\sqrt{\mu_r \varepsilon_r}}$

Also, then use:

$$\omega_0 = \frac{k_0}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \left(\frac{\pi}{L} \frac{1}{\sqrt{\mu_r \varepsilon_r}} \right)$$

This eliminates k_0 and ω_0 and leaves only the patch dimensions.

CAD Formula for Q_{sp} (cont.)

$$Q_{sp} \approx \frac{2\omega_0\left(\frac{1}{8}hWL\right)\mu_0\mu_r}{p\left(\frac{2}{\pi}WL\right)^2 (k_0h)^2\, k_0^2\left(\frac{\eta_0}{6\pi}\right)\mu_r^2 c_1} = \left(\frac{2\cdot 6\pi}{8}\right)\left(\frac{\pi^2}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{1}{WL}\right)\left(\frac{k_0h}{(k_0h)^2}\right)\left(\frac{1}{k_0^3}\right)\left(\frac{\mu_0}{\eta_0}\right)\omega_0$$

$$= \left(\frac{2\cdot 6}{8}\right)\left(\frac{\pi^3}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{1}{WL}\right)\left(\frac{1}{k_0h}\right)\left(\frac{\mu_0}{\eta_0}\right)\left(\frac{1}{k_0^3}\right)(\omega_0)$$

$$= \left(\frac{2\cdot 6}{8}\right)\left(\frac{\pi^3}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{1}{WL}\right)\left(\frac{1}{k_0h}\right)\left(\frac{\mu_0}{\eta_0}\right)\left(\frac{L}{\pi}\sqrt{\mu_r\varepsilon_r}\right)^3\left(\frac{1}{\sqrt{\mu_0\varepsilon_0}}\left(\frac{\pi}{L}\frac{1}{\sqrt{\mu_r\varepsilon_r}}\right)\right)$$

$$k_0 = \frac{\pi}{L}\frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

$$= \left(\frac{2\cdot 6}{8}\right)\left(\frac{\pi^3}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{L}{W}\right)\left(\frac{1}{k_0h}\right)\left(\sqrt{\cancel{\mu_0}\varepsilon_0}\right)\left(\frac{1}{\pi}\sqrt{\mu_r\varepsilon_r}\right)^3\left(\frac{1}{\sqrt{\cancel{\mu_0}\varepsilon_0}}\left(\pi\frac{1}{\sqrt{\mu_r\varepsilon_r}}\right)\right)$$

$$\omega_0 = \frac{k_0}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\left(\frac{\pi}{L}\frac{1}{\sqrt{\mu_r\varepsilon_r}}\right)$$

$$= \left(\frac{2\cdot 6}{8}\right)\left(\frac{\pi^3}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{L}{W}\right)\left(\frac{1}{k_0h}\right)\left(\frac{1}{\pi}\right)^3\left(\mu_r\varepsilon_r\sqrt{\mu_r\varepsilon_r}\right)\left(\pi\frac{1}{\sqrt{\mu_r\varepsilon_r}}\right)$$

CAD Formula for Q_{sp} (cont.)

Continuing with the simplification:

$$Q_{\mathrm{sp}} \approx \left(\frac{2 \cdot 6}{8}\right)\left(\frac{\pi^3}{4}\right)\left(\frac{1}{pc_1}\right)\left(\frac{1}{\mu_r}\right)\left(\frac{L}{W}\right)\left(\frac{1}{k_0 h}\right)\left(\frac{1}{\pi}\right)^3\left(\mu_r \varepsilon_r \sqrt{\mu_r \varepsilon_r}\right)\left(\pi \frac{1}{\sqrt{\mu_r \varepsilon_r}}\right)$$

$$= \left(\frac{3}{8}\right)\left(\frac{1}{pc_1}\right)\left(\frac{L}{W}\right)\left(\frac{1}{k_0 h}\right)(\varepsilon_r)(\pi)$$

$$= \left(\frac{3\pi}{8}\right)\left(\frac{1}{pc_1}\right)\left(\frac{L}{W}\right)\left(\frac{1}{k_0 h}\right)(\varepsilon_r)$$

CAD Formula for Q_{sp} (cont.)

The final approximate CAD formula is:

$$Q_{\text{sp}} \approx \frac{3\pi}{8}\varepsilon_r \left(\frac{L}{W}\right)\left(\frac{1}{k_0 h}\right)\left(\frac{1}{c_1}\right)\left(\frac{1}{p}\right)$$

or

$$\frac{1}{Q_{\text{sp}}} \approx \frac{8}{3\pi}(k_0 h)\left(\frac{W}{L}\right)\left(\frac{1}{\varepsilon_r}\right)c_1 p$$

Recall: $\text{BW} = \frac{1}{\sqrt{2}Q}$

$$\frac{1}{Q} = \frac{1}{Q_{\text{sp}}} + \frac{1}{Q_{\text{sw}}} + \frac{1}{Q_c} + \frac{1}{Q_d}$$

CAD Formula for Q_{sp} (cont.)

Recall that (from Notes 13):

$$p \approx 1 + \frac{a_2}{10}(k_0 W)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)(k_0 W)^4 + c_2\left(\frac{1}{5}\right)(k_0 L)^2 + a_2 c_2 \left(\frac{1}{70}\right)(k_0 W)^2 (k_0 L)^2$$

where

$$a_2 = -0.16605$$
$$a_4 = 0.00761$$
$$c_2 = -0.0914153$$

Also, recall that

$$c_1 \equiv 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$