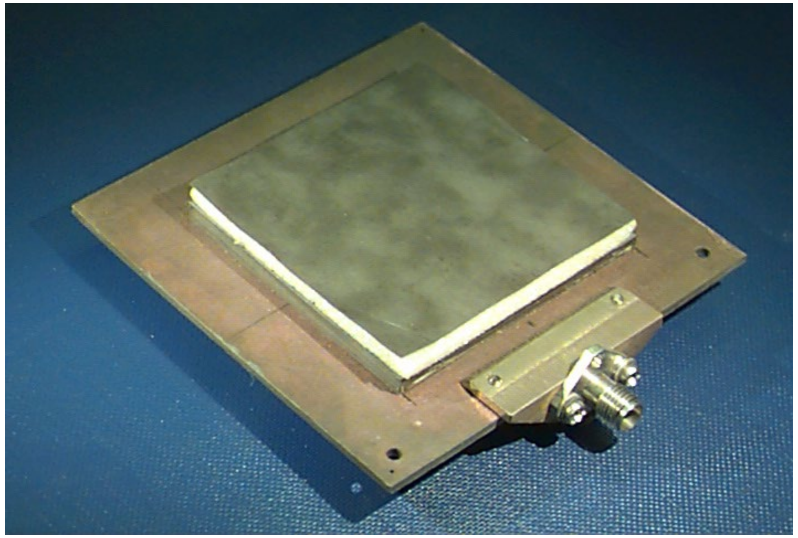
ECE 6345

Spring 2024

Prof. David R. Jackson
ECE Dept.



Notes 16

Overview

In this set of notes we calculate CAD formulas for the directivity, gain, and efficiency of the rectangular patch antenna.

Directivity

Definition of directivity:

$$D(\theta,\phi) \equiv \frac{S_r(r,\theta,\phi)}{\left(\dfrac{P_{\text{sp}}}{4\pi r^2}\right)} = \frac{4\pi r^2 S_r(r,\theta,\phi)}{P_{\text{sp}}}$$

For typical substrate thicknesses, we usually have: $D_{\max} = D(0,0)$

Note:
The angle ϕ is actually arbitrary when $\theta = 0$, but we choose $\phi = 0$.

where

$$D(0,0) = \frac{4\pi r^2 S_r(r,0,0)}{P_{\text{sp}}}$$

$$E_\theta(r,\theta,\phi) = (Il)\,E_0 \cos\phi\; G(\theta)$$

$$E_\phi(r,\theta,\phi) = (Il)\,E_0 \left(-\sin\phi\right) F(\theta)$$

Directivity (cont.)

Recall:

$$(Il)_{\text{patch}} = \left(\frac{2}{\pi} WL\right)$$

$$J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right)$$

The space-wave radiated power of the patch is (from Notes 12):

$$P_{\text{sp}} = p\, P_{\text{sp}}^{\text{dip}}$$

The radiated power density from the patch in the far field is:

$$\begin{aligned} S_r\left(r,0,0\right) &= S_r^{\text{hex}}\left(r,0,0\right)\left|A(0,0)\right|^2 \\ &= S_r^{\text{hex}}\left(r,0,0\right)\left|(Il)_{\text{patch}}\right|^2 \\ &= S_r^{\text{dip}}(r,0,0) \end{aligned}$$

Note: hex denotes a unit-amplitude dipole in the x direction.

The patch and the equivalent dipole have the same power density at broadside.

We then have:

$$D(0,0) = \frac{4\pi r^2\, S_r\left(r,0,0\right)}{P_{\text{sp}}} = \frac{4\pi r^2 S_r^{\text{dip}}(r,0,0)}{pP_{\text{sp}}^{\text{dip}}} = \frac{1}{p}\left(\frac{4\pi r^2 S_r^{\text{dip}}(r,0,0)}{P_{\text{sp}}^{\text{dip}}}\right)$$

Directivity (cont.)

Hence

$$D(0,0)=\frac{1}{p}D^{\text{dip}}(0,0)$$

We next calculate the directivity of the dipole:

$$D^{\text{dip}}(0,0)=\frac{4\pi r^2 S_r^{\text{dip}}(r,0,0)}{P_{\text{sp}}^{\text{dip}}}$$

From previous calculations in Notes 12:

$$P_{\text{sp}}^{\text{dip}}=(Il)^2 k_0^2\left(\frac{\eta_0}{32\pi}\right)\int_0^{\pi/2}\left[|F(\theta)|^2+|G(\theta)|^2\right]\sin\theta\,d\theta\approx(Il)^2(k_0h)^2k_0^2\left(\frac{\eta_0}{6\pi}\right)\mu_r^2c_1$$

$$c_1\equiv 1-\frac{1}{n_1^2}+\frac{2/5}{n_1^4}$$

$$n_1\equiv\sqrt{\varepsilon_r\mu_1}$$

Directivity (cont.)

We then need

$$S_r^{\text{dip}}\left(r,0,0\right)=\frac{1}{2\eta_0}\left[\left|E_\theta^{\text{dip}}\left(0,0\right)\right|^2+\left|E_\phi^{\text{dip}}\left(0,0\right)\right|^2\right]$$

where

$$E_\theta(r,\theta,\phi)=(Il)\,E_0\cos\phi\;G(\theta)$$
$$E_\phi(r,\theta,\phi)=(Il)\,E_0\left(-\sin\phi\right)F(\theta)$$

so that (with $\phi=0$):

$$E_\theta(r,0,0)=(Il)\,E_0\,G(0)$$
$$E_\phi(r,0,0)=0$$

Directivity (cont.)

Hence, we have:

$$S_r^{\mathrm{dip}}(r,0,0) = \frac{1}{2\eta_0} |Il|^2 \, |E_0|^2 \, |G(0)|^2$$

where

$$G(\theta) = \frac{2\cos\theta}{1 - j\left(\dfrac{N_1(\theta)\sec\theta}{\mu_r}\right)\cot\left(k_0 h N_1(\theta)\right)}$$

so

$$G(0) = \frac{2}{1 - j\left(\dfrac{n_1}{\mu_r}\right)\cot\left(k_1 h\right)}$$

Note:

$$N_1(\theta) \equiv \sqrt{n_1^2 - \sin^2\theta}$$

$$k_0 h \, N_1(\theta) = k_0 h \sqrt{n_1^2 - \sin^2\theta}$$

$$N_1(0) = n_1$$

$$k_0 h \, N_1(0) = k_0 h \, n_1 = k_1 h$$

Directivity (cont.)

We then have:

$$S_r^{\text{dip}}(r,0,0)=\frac{1}{2\eta_0}|Il|^2\left(\frac{\omega\mu_0}{4\pi r}\right)^2\frac{4}{1+\left(\dfrac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}$$

We can re-write this using: $\omega\mu_0=k_0\eta_0$

$$S_r^{\text{dip}}(r,0,0)=\eta_0|Il|^2 k_0^2\left(\frac{1}{4\pi r}\right)^2\left(\frac{2}{1+\left(\dfrac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}\right)$$

Directivity (cont.)

Recall:

$$D(0,0) = \frac{1}{p} D^{\text{dip}}(0,0)$$

To summarize so far, we have for the dipole:

$$D^{\text{dip}}(0,0) = \frac{4\pi r^2 S_r^{\text{dip}}(r,0,0)}{P_{\text{sp}}^{\text{dip}}}$$

with

$$S_r^{\text{dip}}(r,0,0) = \eta_0 \left|Il\right|^2 k_0^2 \left(\frac{1}{4\pi r}\right)^2 \left(\frac{2}{1+\left(\frac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}\right)$$

$$P_{\text{sp}}^{\text{dip}} \approx \left|Il\right|^2 (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi}\right) \mu_r^2 c_1$$

Directivity (cont.)

We then have:

$$D^{\mathrm{dip}}(0,0) \approx \frac{4\pi r^2 \eta_0 |Il|^2 k_0^2 \left(\dfrac{1}{4\pi r}\right)^2 \left(\dfrac{2}{1+\left(\dfrac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}\right)}{|Il|^2 (k_0 h)^2 k_0^2 \left(\dfrac{\eta_0}{6\pi}\right)\mu_r^2 c_1}$$

Directivity (cont.)

The result is:

$$D^{\mathrm{dip}}(0,0) \approx 3\left(\frac{1}{k_0 h}\right)^2 \left(\frac{1}{\mu_r^2 c_1}\right)\left[\frac{1}{1+\dfrac{\varepsilon_r}{\mu_r}\cot^2(k_1 h)}\right]$$

This may be re-written as (multiplying and dividing by a tangent squared term):

$$D^{\mathrm{dip}}(0,0) \approx 3\left(\frac{\tan(k_1 h)}{k_1 h}\right)^2 \left(\frac{k_1}{k_0}\right)^2 \left(\frac{1}{\mu_r^2 c_1}\right)\left[\frac{1}{\tan^2(k_1 h)+\dfrac{\varepsilon_r}{\mu_r}}\right]$$

Directivity (cont.)

or

$$D^{\text{dip}}(0,0) \approx 3\left(\frac{\tan(k_1 h)}{k_1 h}\right)^2 \left(\frac{\varepsilon_r}{\mu_r c_1}\right)\left[\frac{1}{\tan^2(k_1 h) + \dfrac{\varepsilon_r}{\mu_r}}\right]$$

or

$$D^{\text{dip}}(0,0) \approx 3\,\text{tanc}^2\left(k_1 h\right)\left(\frac{1}{c_1}\right)\left[\frac{1}{1 + \dfrac{\mu_r}{\varepsilon_r}\tan^2(k_1 h)}\right]$$

where $\text{tanc}\left(x\right) \equiv \dfrac{\tan\left(x\right)}{x}$

Directivity (cont.)

Since the substrate is assumed to be thin, we can further approximate this as:

$$D^{\text{dip}}(0,0) \approx \frac{3}{c_1}$$

Recall:

$$D(0,0) = \frac{1}{p} D^{\text{dip}}(0,0)$$

where

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

Note: for $n_1 >> 1, \; D^{\text{dip}}(0,0) \approx 3$ $\quad \left(D^{\text{dip}}(0,0) \approx 4.77 \text{ dB} \right)$

Gain

The gain of the patch is related to the directivity as:

$$G(0,0) = D(0,0)\, e_r$$

where

$$e_r = \frac{Q}{Q_{\mathrm{sp}}}$$

Note:
CAD formulas for all Q's have now been derived, except for Q_{sw}.

and

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{\mathrm{sp}}} + \frac{1}{Q_{\mathrm{sw}}}$$

CAD formulas for all of the Q factors were presented in Notes 3.

Gain (cont.)

From Notes 1:

$$Q_{\text{sw}} = Q_{\text{sp}} \left(\frac{e_r^{\text{sw}}}{1 - e_r^{\text{sw}}} \right)$$

$$e_r^{\text{sw}} \approx e_r^{\text{hed}}$$

$$e_r^{\text{hed}} \approx \frac{1}{1 + (k_0 h) \left(\frac{3\pi}{4} \right) \mu_r \frac{1}{c_1} \left(1 - \frac{1}{n_1^2} \right)^3}$$

(This will be derived later from the spectral-domain method.)

Summary

$$G(0,0) = D(0,0)\, e_r$$

$$D(0,0) \approx \frac{3}{p c_1}$$

$$e_r = \frac{Q}{Q_{\text{sp}}}$$

$$p = 1 + \frac{a_2}{10}(k_0 W)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)(k_0 W)^4 + c_2\left(\frac{1}{5}\right)(k_0 L)^2 + a_2 c_2\left(\frac{1}{70}\right)(k_0 W)^2 (k_0 L)^2$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{\text{sp}}} + \frac{1}{Q_{\text{sw}}}$$

$$Q_{\text{sp}} \approx \frac{3\pi}{8}\varepsilon_r\left(\frac{L}{W}\right)\left(\frac{1}{k_0 h}\right)\left(\frac{1}{c_1}\right)\left(\frac{1}{p}\right)$$

$$Q_c = \left(\frac{\eta_0}{2}\right)\mu_r\left[\frac{(k_0 h)}{R_s^{\text{ave}}}\right]$$

$$Q_d = \frac{1}{\tan\delta}$$

$$Q_{\text{sw}} = Q_{\text{sp}}\left(\frac{e_r^{\text{sw}}}{1 - e_r^{\text{sw}}}\right)$$

$$e_r^{\text{sw}} \approx e_r^{\text{hed}}$$

$$e_r^{\text{hed}} \approx \frac{1}{1 + (k_0 h)\left(\frac{3\pi}{4}\right)\mu_r \frac{1}{c_1}\left(1 - \frac{1}{n_1^2}\right)^3}$$