ECE 6345

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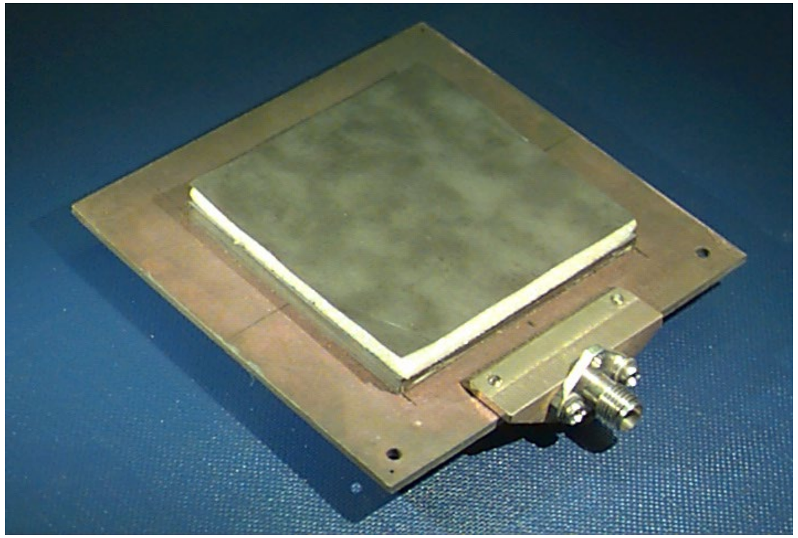

Notes 17

Overview

- In this set of notes we calculate the power radiated into space by the circular patch.

- This will lead to Q_{sp} of the circular patch (in Notes 20).

Radiated Power of Circular Patch

From Notes 11 we have:

$$E_{\theta}^{\mathrm{FF}}\left(r,\theta,\phi\right)=\frac{E_0}{\eta_0}(ah)\cos\phi\,\mathrm{tanc}(k_{z1}h)\,Q(\theta)\;2\pi\,J_1'(k_0 a\sin\theta)$$

$$E_{\phi}^{\mathrm{FF}}\left(r,\theta,\phi\right)=-\frac{E_0}{\eta_0}(ah)\sin\phi\,\mathrm{tanc}\left(k_{z1}h\right)\,P(\theta)\;2\pi J_{\mathrm{inc}}(k_0 a\sin\theta)$$

Assumption:

$$E_z\left(a,\phi\right)=\cos\phi$$

$$J_{\mathrm{inc}}(x)\equiv\frac{J_1(x)}{x}$$

where $P(\theta)\equiv\cos\theta\left(1-\Gamma^{\mathrm{TE}}(\theta)\right)$ $\qquad Q(\theta)=1-\Gamma^{\mathrm{TM}}(\theta)$

$$1-\Gamma^{\mathrm{TM}}\left(\theta\right)=\frac{2}{1+j\left(\dfrac{N_1(\theta)\sec\theta}{\varepsilon_r}\right)\tan\left(k_0 h\,N_1(\theta)\right)}$$

$$1-\Gamma^{\mathrm{TE}}\left(\theta\right)=\frac{2}{1+j\left(\dfrac{\mu_r\cos\theta}{N_1(\theta)}\right)\tan\left(k_0 h\,N_1(\theta)\right)}$$

$$E_0=\left(\frac{-j\omega\mu_0}{4\pi r}\right)e^{-jk_0 r}$$

$$k_{z1}=k_0 N_1\left(\theta\right)$$

$$N_1\left(\theta\right)\equiv\sqrt{n_1^2-\sin^2\theta}$$

$$n_1=\sqrt{\mu_r\varepsilon_r}$$

Radiated Power of Circular Patch (cont.)

The power density in the far field from the Poynting vector is

$$S_r(r,\theta,\phi) = \frac{1}{2\eta_0}\left[\left|E_\theta\right|^2 + \left|E_\phi\right|^2\right]$$

$$= \frac{1}{2\eta_0}\frac{\left|E_0\right|^2}{\eta_0^2}(a\,h)^2 \operatorname{tanc}^2(k_{z1}\,h)(2\pi)^2$$

$$\cdot\left[\cos^2\phi\, J_1'^2(k_0 a\sin\theta)\left|Q(\theta)\right|^2 + \sin^2\phi\, J_{\text{inc}}^2(k_0 a\sin\theta)\left|P(\theta)\right|^2\right]$$

Next, use

$$\left|E_0\right| = \frac{\omega\,\mu_0}{4\pi\,r} = \frac{k_0\,\eta_0}{4\pi\,r}$$

$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi r}\right)e^{-jk_0 r}$$

Radiated Power of Circular Patch (cont.)

We then have:

$$S_r(r,\theta,\phi) = \frac{1}{8\eta_0}(k_0 a)^2 h^2 \,\mathrm{tanc}^2(k_{z1} h)$$

$$\cdot\left[\cos^2\phi \left|Q(\theta)\right|^2 J_1'^2(k_0 a\sin\theta) + \sin^2\phi\left|P(\theta)\right|^2 J_{\mathrm{inc}}^2(k_0 a \sin\theta)\right]\frac{1}{r^2}$$

The space-wave power is then

$$P_{\mathrm{sp}} = \int_0^{2\pi}\int_0^{\pi/2} S_r(r,\theta,\phi)\, r^2 \sin\theta \, d\theta\, d\phi$$

Performing the ϕ integrals, we have:

$$P_{\mathrm{sp}} = \pi\frac{1}{8\eta_0}(k_0 a)^2 h^2 \int_0^{\pi/2} \mathrm{tanc}^2(k_{z1} h)\left[\left|Q(\theta)\right|^2 J_1'^2(k_0 a\sin\theta) + \left|P(\theta)\right|^2 J_{\mathrm{inc}}^2(k_0 a\sin\theta)\right]\sin\theta\, d\theta$$

Radiated Power of Circular Patch (cont.)

Define $$I_c \equiv \int_0^{\pi/2} C(\theta)\, d\theta$$

where

$$C(\theta) = \mathrm{tanc}^2\left(k_{z1}\, h\right)\left[\left|Q(\theta)\right|^2 J_1'^2\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\mathrm{inc}}^2\left(k_0 a \sin\theta\right)\right]\sin\theta$$

We then have:

$$P_{\mathrm{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 I_c$$

Note: We will get a CAD formula for I_c later.