ECE 6345

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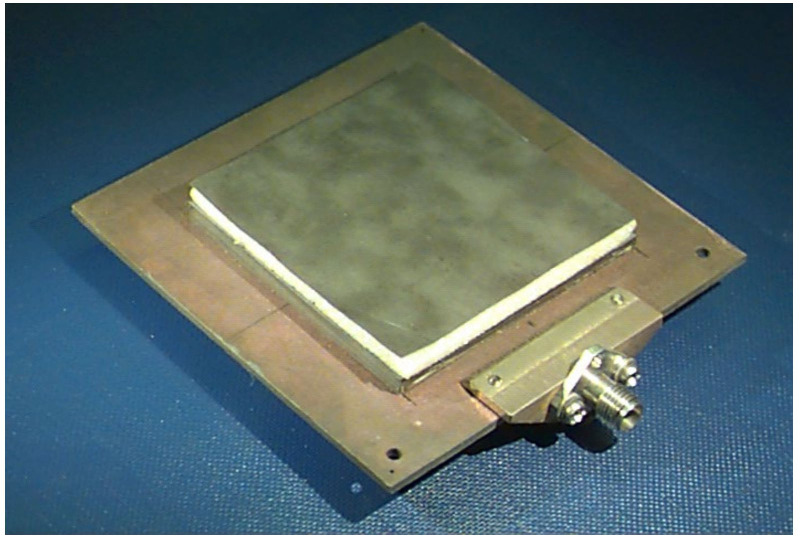


Notes 18

Overview

❖ In this set of notes we introduce the p factor (called p_c) of the circular patch.

Radiated Power of Circular Patch (cont.)

From Notes 17:

$$P_{\text{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 I_c$$

where

$$C(\theta) = \text{tanc}^2\left(k_{z1} h\right)\left[\left|Q(\theta)\right|^2 J_1'^2\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\text{inc}}^2\left(k_0 a \sin\theta\right)\right]\sin\theta$$

$$I_c \equiv \int_0^{\pi/2} C(\theta)\, d\theta$$

Note: We will get a CAD formula for I_c later.

The p_c Factor

Here we define the p factor for the circular patch (called p_c).

$$p_c \equiv \frac{I_c}{I_0} = \frac{\int_0^{\pi/2} C(\theta)\,d\theta}{\int_0^{\pi/2} C_0(\theta)\,d\theta}$$

$\Rightarrow$

$$P_{\mathrm{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 I_0 p_c$$

$$I_0 \equiv \int_0^{\pi/2} C_0(\theta)\,d\theta$$

$$C_0(\theta) \equiv C(\theta)\,\Big|_{a \to 0}$$

The term C_0 ignores the patch space (array) factor.

The p_c term gives the ratio of the power radiated by the actual patch to the power radiated if we ignore the array factor, and collapse the magnetic current down to a single magnetic dipole.

The p_c Factor

Note that as $x \to 0$

$$J_1'(x) \to \frac{1}{2}$$

$$J_{\text{inc}}(x) \to \frac{1}{2}$$

Hence, we have:

$$C(\theta) = \sin\theta \ \text{tanc}^2\left(k_{z1}\, h\right)\left[\left|Q(\theta)\right|^2 J_1'^2\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\text{inc}}^2\left(k_0 a \sin\theta\right)\right]$$

$$\Downarrow$$

$$C_0(\theta) = \sin\theta \ \text{tanc}^2\left(k_{z1}\, h\right)\left[\left|Q(\theta)\right|^2 \frac{1}{4} + \left|P(\theta)\right|^2 \frac{1}{4}\right]$$

This allows us to express I_0 in a simple form without the Bessel functions:

$$I_0 = \int_0^{\pi/2} \sin\theta \, \text{tanc}^2\left(k_0 h\, N_1(\theta)\right)\frac{1}{4}\left[\left|P(\theta)\right|^2 + \left|Q(\theta)\right|^2\right] d\theta$$

The p_c Factor (cont.)

For the p_c factor we then have:

$$p_c = \frac{\displaystyle\int_0^{\pi/2} \sin\theta \, \mathrm{tanc}^2\left(k_0 h \, N_1(\theta)\right)\left[\left|Q(\theta)\right|^2 J_1'^2\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\mathrm{inc}}^2\left(k_0 a \sin\theta\right)\right] d\theta}{\displaystyle\int_0^{\pi/2} \sin\theta \, \mathrm{tanc}^2\left(k_0 h \, N_1(\theta)\right)\frac{1}{4}\left[\left|P(\theta)\right|^2 + \left|Q(\theta)\right|^2\right] d\theta}$$

The term p depends on the patch radius a and the substrate parameters.
(After making some approximations, it will depend only on the patch radius.)

Approximation for a Thin Substrate

For a thin substrate, we have:

$$\operatorname{tanc}^2\left(k_0 h\, N_1(\theta)\right) \approx 1$$

so

$$I_0 \approx \int_0^{\pi/2} \sin\theta \,\frac{1}{4}\left[\left|P(\theta)\right|^2 + \left|Q(\theta)\right|^2\right] d\theta$$

$$p_c \approx \frac{\displaystyle\int_0^{\pi/2} \sin\theta \left[\left|Q(\theta)\right|^2 J_1'^2\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\text{inc}}^2\left(k_0 a \sin\theta\right)\right] d\theta}{\displaystyle\int_0^{\pi/2} \sin\theta \,\frac{1}{4}\left[\left|P(\theta)\right|^2 + \left|Q(\theta)\right|^2\right] d\theta}$$

Approximation for a Thin Substrate (cont.)

We also have:

$$P(\theta) = \cos\theta\,(1 - \Gamma^{\mathrm{TE}}(\theta)) = \frac{2\cos\theta}{1 + j\left(\dfrac{\mu_r \cos\theta}{N_1(\theta)}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$

$$Q(\theta) = 1 - \Gamma^{\mathrm{TM}}(\theta) = \frac{2}{1 + j\left(\dfrac{N_1(\theta)\sec\theta}{\varepsilon_r}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$

For a thin substrate we then have

$$P(\theta) \approx 2\cos\theta$$

$$Q(\theta) \approx 2$$

Approximation for a Thin Substrate (cont.)

For the I_0 term (the denominator of the p_c function) we then have:

$$I_0 \approx \int_0^{\pi/2} \sin\theta \, \frac{1}{4}\left[\left|P(\theta)\right|^2 + \left|Q(\theta)\right|^2\right] d\theta$$

$$I_0 \approx \int_0^{\pi/2} \frac{1}{4}\sin\theta \, \left[4\left(\cos^2\theta + 1\right)\right] d\theta$$

$$= \int_0^{\pi/2} \sin\theta \, \left(\cos^2\theta + 1\right) d\theta$$

This yields

$$I_0 \approx \frac{4}{3}$$

Approximation for a Thin Substrate (cont.)

The formula for the p_c function then becomes:

$$p \approx \frac{3}{4} \int_0^{\pi/2} \sin\theta \left[|2|^2 J_1'^2\left(k_0 a \sin\theta\right) + |2\cos\theta|^2 J_{\mathrm{inc}}^2\left(k_0 a \sin\theta\right) \right] d\theta$$

so that

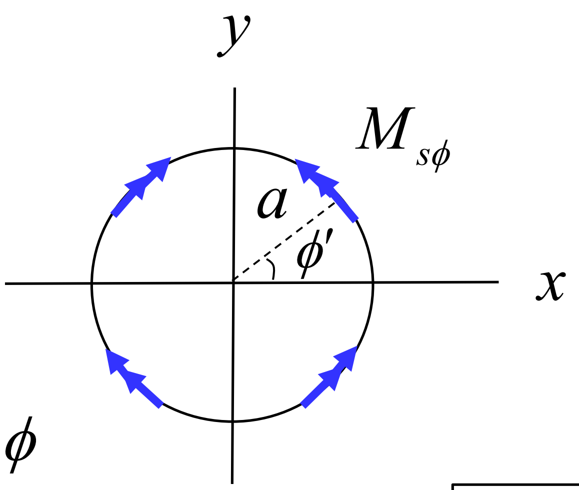
$$p \approx 3 \int_0^{\pi/2} \sin\theta \left[J_1'^2\left(k_0 a \sin\theta\right) + \cos^2\theta \; J_{\mathrm{inc}}^2\left(k_0 a \sin\theta\right) \right] d\theta$$

The p factor now only depends only on the patch size.

Equivalent Dipole Moment of Circular Patch

Consider an equivalent magnetic dipole that models the patch ($a \to 0$):

Patch



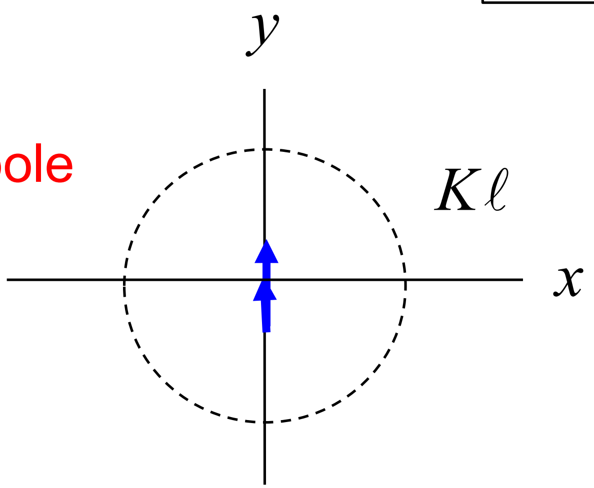
$M_{s\phi} = \cos\phi$

$h / \lambda_0 \ll 1$

$E_z(a,\phi) = \cos\phi$

As $a \to 0$ the magnetic current sheet at the edge approaches an equivalent magnetic dipole.

Equivalent dipole



Equivalent Dipole Moment of Circular Patch (cont.)

The dipole moment of the equivalent magnetic dipole is calculated:

$$Kl = \int_S \underline{M}_s \cdot \underline{\hat{y}} \, dS = h \int_0^{2\pi} M_{s\phi} \cos\phi \, a \, d\phi = h \int_0^{2\pi} \cos\phi \cos\phi \, a d\phi =$$

$$ha \int_0^{2\pi} \cos^2 \phi \, a d\phi$$

Note : $\underline{\hat{\phi}} \cdot \underline{\hat{y}} = \cos\phi$

This yields $Kl = \pi a h$

Equivalent Dipole Moment of Circular Patch (cont.)

We can therefore physically interpret the p_c factor as follows:

$$p_c = \frac{P_{\text{rad}}^{\text{patch}}}{P_{\text{rad}}^{\text{dip}}}$$

where

$P_{\text{rad}}^{\text{patch}}$ = power radiated by circular patch

$P_{\text{rad}}^{\text{dip}}$ = power radiated by magnetic dipole of equal moment $(Kl = \pi a h)$

CAD Formula for p_c

In the next set of notes we will obtain an approximate closed-form CAD expression for p_c.