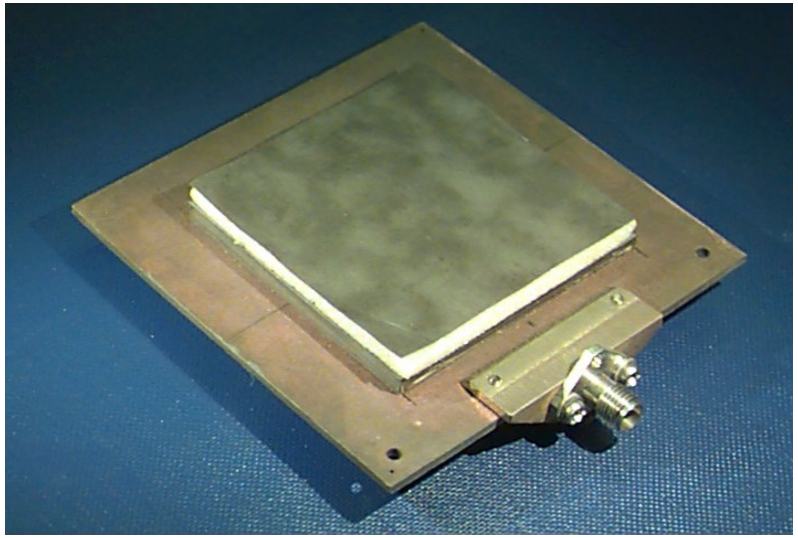
ECE 6345

Spring 2024

Prof. David R. Jackson
ECE Dept.



Notes 20

Overview

❖ In this set of notes we find a CAD formula for Q_{sp} of the circular patch.

Radiated Power of Circular Patch (cont.)

From Notes 18:

$$P_{\mathrm{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 \, I_c \qquad I_c = I_0 p_c$$

$$I_0 \approx \frac{4}{3} \qquad p_c \approx 3\int_0^{\pi/2} \sin\theta \left[J_1'^2\left(k_0 a \sin\theta\right) + \cos^2\theta \; J_{\mathrm{inc}}^2\left(k_0 a \sin\theta\right)\right] d\theta$$

From Notes 19:

$$p_c \approx \sum_{k=0}^{6} \left(k_0 a\right)^{2k} \, e_{2k}$$

where

$$\begin{aligned}
e_0 &= 1 \\
e_2 &= -0.400000 \\
e_4 &= 0.0785710 \\
e_6 &= -7.27509 \times 10^{-3} \\
e_8 &= 3.81786 \times 10^{-4} \\
e_{10} &= -1.09839 \times 10^{-5} \\
e_{12} &= 1.47731 \times 10^{-7}
\end{aligned}$$

Calculation of Q_{sp}

The Q formula is $$Q_{\text{sp}} = \omega_0 \left(\frac{U_S}{P_{\text{sp}}} \right)$$

$$\begin{aligned}
U_S &= 2U_E \\
&= 2\int_V \frac{1}{4}\varepsilon_0\,\varepsilon_r \left|E_z\right|^2 dV \\
&= \frac{1}{2}\varepsilon_0\varepsilon_r h \int_S \left|E_z\right|^2 dS \\
&= \frac{1}{2}\varepsilon_0\varepsilon_r h \int_0^{2\pi}\int_0^{a} \left|E_z(\rho,\phi)\right|^2 \rho\, d\rho\, d\phi \\
&= \pi \frac{1}{2}\varepsilon_0\varepsilon_r h \int_0^{a} \left|E_z(\rho,0)\right|^2 \rho\, d\rho
\end{aligned}$$

The electric field inside the patch cavity is:

$$E_z(\rho,\phi) = \cos\phi \left[\frac{J_1(k\rho)}{J_1(ka)} \right]$$

Note:

$$E_z(a,\phi) = \cos\phi$$

$$\left(E_z(\rho,\phi) = E_z(\rho,0)\cos\phi \right)$$

Calculation of Q_{sp} (cont.)

The stored energy is then:

$$U_S = \frac{1}{2}\varepsilon_0\varepsilon_r h\pi \frac{1}{J_1^2(ka)} \int_0^a J_1^2(k\rho)\,\rho\, d\rho$$

Denote:

$$I_1 = \int_0^a J_1^2(k\rho)\,\rho\, d\rho = \left[\frac{1}{2}\rho^2 J_1'^2(k\rho) + \frac{1}{2}\rho^2\left(1-\frac{1}{(k\rho)^2}\right)J_1^2(k\rho)\right]_0^a$$

$$= \frac{a^2}{2}\left[J_1'^2(ka) + \left(1-\frac{1}{(ka)^2}\right)J_1^2(ka)\right]$$

Calculation of Q_{sp} (cont.)

Recall that $k = \dfrac{x'_{11}}{a}$ $\quad x'_{11} = 1.84118$

so $J'_1(ka) = J'_1(x'_{11}) = 0$

Hence $I_1 = \dfrac{a^2}{2}\left(1 - \dfrac{1}{x_{11}^2}\right) J_1^2(x'_{11})$

We then have:

$$U_S = \frac{1}{2}\varepsilon_0\varepsilon_r h\,\pi\left(\frac{1}{J_1^2(x'_{11})}\right)\left[\left(\frac{a^2}{2}\right)\left(1 - \frac{1}{x'^2_{11}}\right)J_1^2(x'_{11})\right]$$

or

$$U_S = \frac{1}{2}\varepsilon_0\varepsilon_r h\,\pi\left[\left(\frac{a^2}{2}\right)\left(1 - \frac{1}{x'^2_{11}}\right)\right]$$

Calculation of Q_{sp} (cont.)

We then have:

$$Q_{\mathrm{sp}} = \omega_0 \left(\frac{\frac{1}{2}\varepsilon_0 \varepsilon_r h \pi \left(\frac{a^2}{2}\right)\left(1 - \frac{1}{x_{11}'^2}\right)}{\frac{\pi}{8\eta_0}(k_0 a)^2 h^2 p_c I_0} \right)$$

This may be re-written by using the following expressions to eliminate ω_0 and k_0:

$$k_0 \sqrt{\mu_r \varepsilon_r}\, a = x_{11}'$$
$$\omega_0 \sqrt{\mu_0 \varepsilon_0} \sqrt{\mu_r \varepsilon_r}\, a = x_{11}'$$

$\Rightarrow$

$$k_0 = \frac{x_{11}'}{a\sqrt{\mu_r \varepsilon_r}}$$

$$\omega_0 = \frac{x_{11}'}{a\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r}}$$

Calculation of Q_{sp} (cont.)

We then have:

$$Q_{\mathrm{sp}} = \omega_0 \left(\frac{\frac{1}{2}\varepsilon_0 \varepsilon_r h \pi \left(\frac{1}{2}\right)\left(1 - \frac{1}{x_{11}'^2}\right)}{\frac{\pi}{8\eta_0}(k_0 k_0) h^2 \, p_c I_0} \right) = \frac{x_{11}'}{a\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r}} \left(\frac{\frac{1}{2}\varepsilon_0 \varepsilon_r h \pi \left(\frac{1}{2}\right)\left(1 - \frac{1}{x_{11}'^2}\right)}{\frac{\pi}{8\eta_0} k_0 \left(\frac{x_{11}'}{a\sqrt{\mu_r \varepsilon_r}}\right) h^2 \, p_c I_0} \right)$$

$$Q_{\mathrm{sp}} = \frac{x_{11}'}{a\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r}} \left(\frac{\frac{1}{2}\varepsilon_0 \varepsilon_r h \pi \left(\frac{1}{2}\right)\left(1 - \frac{1}{x_{11}'^2}\right)}{\frac{\pi}{8\eta_0} k_0 \left(\frac{x_{11}'}{a\sqrt{\mu_r \varepsilon_r}}\right) h^2 \, p_c I_0} \right) = \frac{x_{11}'}{\sqrt{\mu_0 \varepsilon_0}} \left(\frac{2\varepsilon_0 \varepsilon_r \left(1 - \frac{1}{x_{11}'^2}\right)}{\frac{1}{\eta_0}(k_0 h)(x_{11}') \, p_c I_0} \right)$$

Calculation of Q_{sp} (cont.)

We then have:

$$Q_{\mathrm{sp}} = \frac{2}{x_{11}'^{2}}\left(x_{11}'^{2}-1\right)\left(\frac{1}{k_0 h}\right)\varepsilon_r\left[\frac{1}{p_c I_0}\right] \qquad I_0 \approx \frac{4}{3}$$

or

$$Q_{\mathrm{sp}} = \frac{3}{2}\frac{1}{x_{11}'^{2}}\left(x_{11}'^{2}-1\right)\left(\frac{1}{k_0 h}\right)\varepsilon_r\left[\frac{1}{p_c}\right]$$

Note that Q_{sp} is proportional to the substrate permittivity and inversely proportional to the substrate thickness.

Approximation for a Thin Substrate (cont.)

Summary

$$Q_{\text{sp}} = \frac{3}{2}\frac{1}{x_{11}'^2}\left(x_{11}'^2 - 1\right)\left(\frac{1}{p_c}\right)\left(\frac{1}{k_0 h}\right)\varepsilon_r$$

$$x_{11}' = 1.84118$$

$$p_c \approx \sum_{k=0}^{6} \left(k_0 a\right)^{2k} e_{2k}$$

where

$$e_0 = 1$$
$$e_2 = -0.400000$$
$$e_4 = 0.0785710$$
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