ECE 6345

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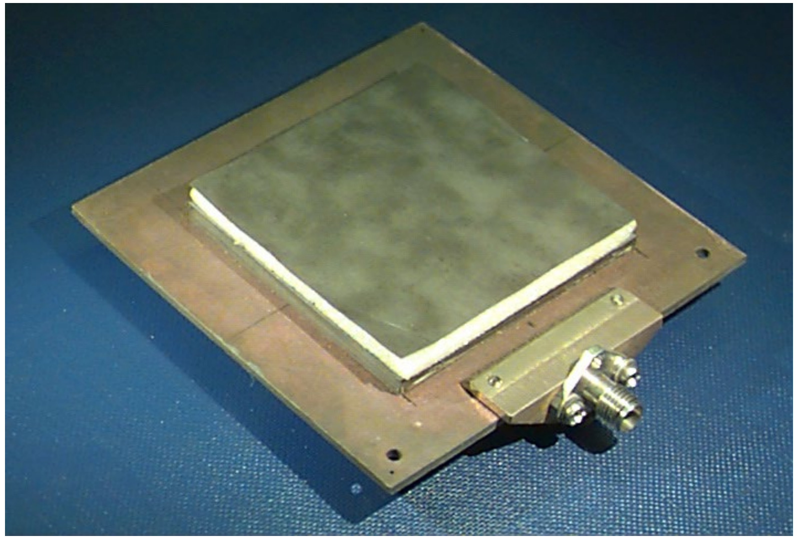
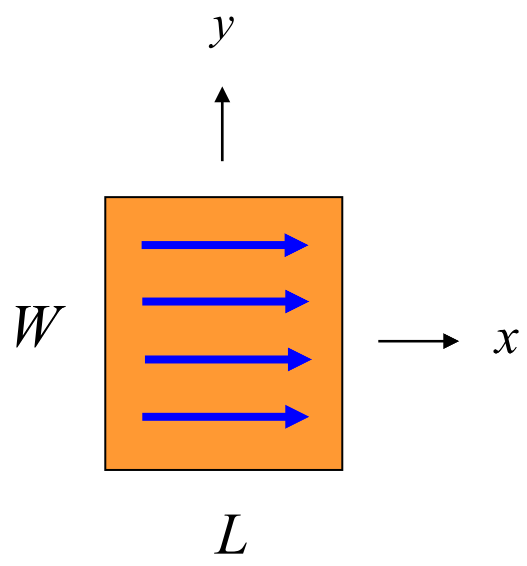


Notes 24

Overview

❖ In this set of notes we use the spectral-domain method to calculate the surface-wave radiation efficiency e_r^{sw} (radiation efficiency due only to surface-wave loss) of a rectangular microstrip antenna.

➢ We assume a lossless substrate and PEC metal in this set of notes.



Overview

From previous derivation:

$$Q_{\mathrm{sw}} = Q_{\mathrm{sp}} \left(\frac{e_r^{\mathrm{sw}}}{1 - e_r^{\mathrm{sw}}} \right) \qquad e_r^{\mathrm{sw}} \equiv \frac{P_{\mathrm{sp}}}{P_{\mathrm{rad}}}$$

P_{rad} = power radiated into space <u>and</u> the surface wave.

Note:

$$e_r = \frac{P_{\mathrm{sp}}}{P_{\mathrm{tot}}} = \left(\frac{P_{\mathrm{sp}}}{P_{\mathrm{sp}} + P_{\mathrm{sw}}} \right) \left(\frac{P_{\mathrm{sp}} + P_{\mathrm{sw}}}{P_{\mathrm{tot}}} \right)$$

$$= \left(\frac{P_{\mathrm{sp}}}{P_{\mathrm{rad}}} \right) \left(\frac{P_{\mathrm{rad}}}{P_{\mathrm{tot}}} \right)$$

$$= e_r^{\mathrm{sw}} \, e_r^{\mathrm{diss}}$$

Total Radiated Power

$$P_{\text{rad}} = P_{\text{sp}} + P_{\text{sw}}$$

$$P_{\text{rad}} = \text{Re}\left\{-\frac{1}{2}\int_S \underline{E}\cdot\underline{J}_s^*\, dS\right\}$$

$$= \text{Re}\left\{-\frac{1}{2}\int_S E_x \cdot J_{sx}^*\, dS\right\}$$

$$= \text{Re}\left\{-\frac{1}{2}\int_S J_{sx}^*(x,y)\left(\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{E}_x\left(k_x,k_y\right)\cdot e^{-j\left(k_x x+k_y y\right)}dk_x dk_y\right)dx\,dy\right\}$$

$$= \text{Re}\left\{-\frac{1}{2}\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\int_S J_{sx}^*(x,y)e^{-j\left(k_x x+k_y y\right)}dx\,dy\right)\tilde{E}_x\left(k_x,k_y\right)dk_x dk_y\right\}$$

(switching the order of integration)

$$= \text{Re}\left\{-\frac{1}{2}\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\tilde{J}_{sx}\left(k_x,k_y\right)\right]^*\tilde{E}_x\left(k_x,k_y\right)dk_x dk_y\right\}$$

Recall:

$$\tilde{J}_{sx}(k_x,k_y) = \int_S J_{sx}(x,y)e^{+j\left(k_x x+k_y y\right)}dx\,dy$$

Total Radiated Power (cont.)

$$P_{\mathrm{rad}} = \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\tilde{J}_{sx}\left(k_x,k_y\right) \right]^* \tilde{E}_x\left(k_x,k_y\right) dk_x dk_y \right\}$$

The transform of the current is a real function of k_x and k_y: $\tilde{J}_{sx}\left(k_x,k_y\right) = \left(\frac{\pi}{2}LW\right)\mathrm{sinc}\left(k_y\frac{W}{2}\right)\left[\frac{\cos\left(k_x\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(k_x\frac{L}{2}\right)^2}\right]$

We then have (dropping the conjugate):

$$P_{\mathrm{rad}} = \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{J}_{sx}\left(k_x,k_y\right) \tilde{E}_x\left(k_x,k_y\right) dk_x dk_y \right\}$$

Note: The transform with the conjugate is not analytic, but the transform without the conjugate is.

Total Radiated Power (cont.)

In *polar coordinates* we have:

$$P_{\text{rad}} = -\frac{1}{8\pi^2}\,\text{Re}\int_0^{2\pi}\int_0^{+\infty} \tilde{E}_x\,\tilde{J}_{sx}\,k_t\,dk_t\,d\bar{\phi}$$

Next, use $\tilde{E}_x = \tilde{G}_{xx}\tilde{J}_{sx}$ so that

$$P_{\text{rad}} = -\frac{1}{8\pi^2}\,\text{Re}\int_0^{2\pi}\int_0^{+\infty} \tilde{G}_{xx}\tilde{J}_{sx}^2\,k_t\,dk_t\,d\bar{\phi}$$

Using symmetry, we have:

$$P_{\text{rad}} = -\frac{1}{2\pi^2}\,\text{Re}\int_0^{\pi/2}\int_0^{+\infty} \tilde{G}_{xx}\tilde{J}_{sx}^2\,k_t\,dk_t\,d\bar{\phi}$$

Define

$$F_p\left(k_t,\bar{\phi}\right) \equiv -\frac{1}{2\pi^2}\tilde{G}_{xx}\tilde{J}_{sx}^2\,k_t$$

Recall:

$$\tilde{G}_{xx} = -\left[\left(\frac{k_x}{k_t}\right)^2 V_i^{\text{TM}}(z) + \left(\frac{k_y}{k_t}\right)^2 V_i^{\text{TE}}(z)\right]$$

Total Radiated Power (cont.)

Then

$$P_{\text{rad}} = \text{Re} \int_0^{\pi/2} \int_0^{+\infty} F_p\left(k_t, \bar{\phi}\right)\, dk_t \, d\bar{\phi}$$

Using the previous "denominator" notation, we have:

$$\tilde{G}_{xx} = -\left[\left(\frac{k_x}{k_t}\right)^2 \frac{1}{D^{\text{TM}}(k_t)} + \left(\frac{k_y}{k_t}\right)^2 \frac{1}{D^{\text{TE}}(k_t)} \right]$$

Note:

$$V_i^{\text{TM}}(z) = \frac{1}{D^{\text{TM}}}$$

$$V_i^{\text{TE}}(z) = \frac{1}{D^{\text{TE}}}$$

$$D^{\text{TM}}(k_t) = Y_0^{\text{TM}} - jY_1^{\text{TM}} \cot\left(k_{z1} h\right) = \left(\frac{\omega \varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega \varepsilon_1}{k_{z1}}\right) \cot\left(k_{z1} h\right)$$

$$D^{\text{TE}}(k_t) = Y_0^{\text{TE}} - jY_1^{\text{TE}} \cot\left(k_{z1} h\right) = \left(\frac{k_{z0}}{\omega \mu_0}\right) - j\left(\frac{k_{z1}}{\omega \mu_1}\right) \cot\left(k_{z1} h\right)$$

Total Radiated Power (cont.)

We have the following properties:

$$k_t < k_0: \quad k_{z0} = \text{real}, \quad k_{z1} = \text{real}, \qquad D^{\text{TM}} = \text{complex}$$

$$k_t > k_0: \quad k_{z0} = \text{imaginary}, \quad k_{z1} = \text{real} \text{ or } \text{imaginary}, \qquad D^{\text{TM}} = \text{imaginary}$$

(The same is true for D^{TE}.)

$$k_{z0} = \left(k_0^2 - k_t^2\right)^{1/2}$$
$$k_{z1} = \left(k_1^2 - k_t^2\right)^{1/2}$$

Note: The correct square root is always obtained by using:

$$k_{z0} = -j\sqrt{k_t^2 - k_0^2}$$
$$k_{z1} = -j\sqrt{k_t^2 - k_1^2}$$

(The radical sign denotes the principal square root.)

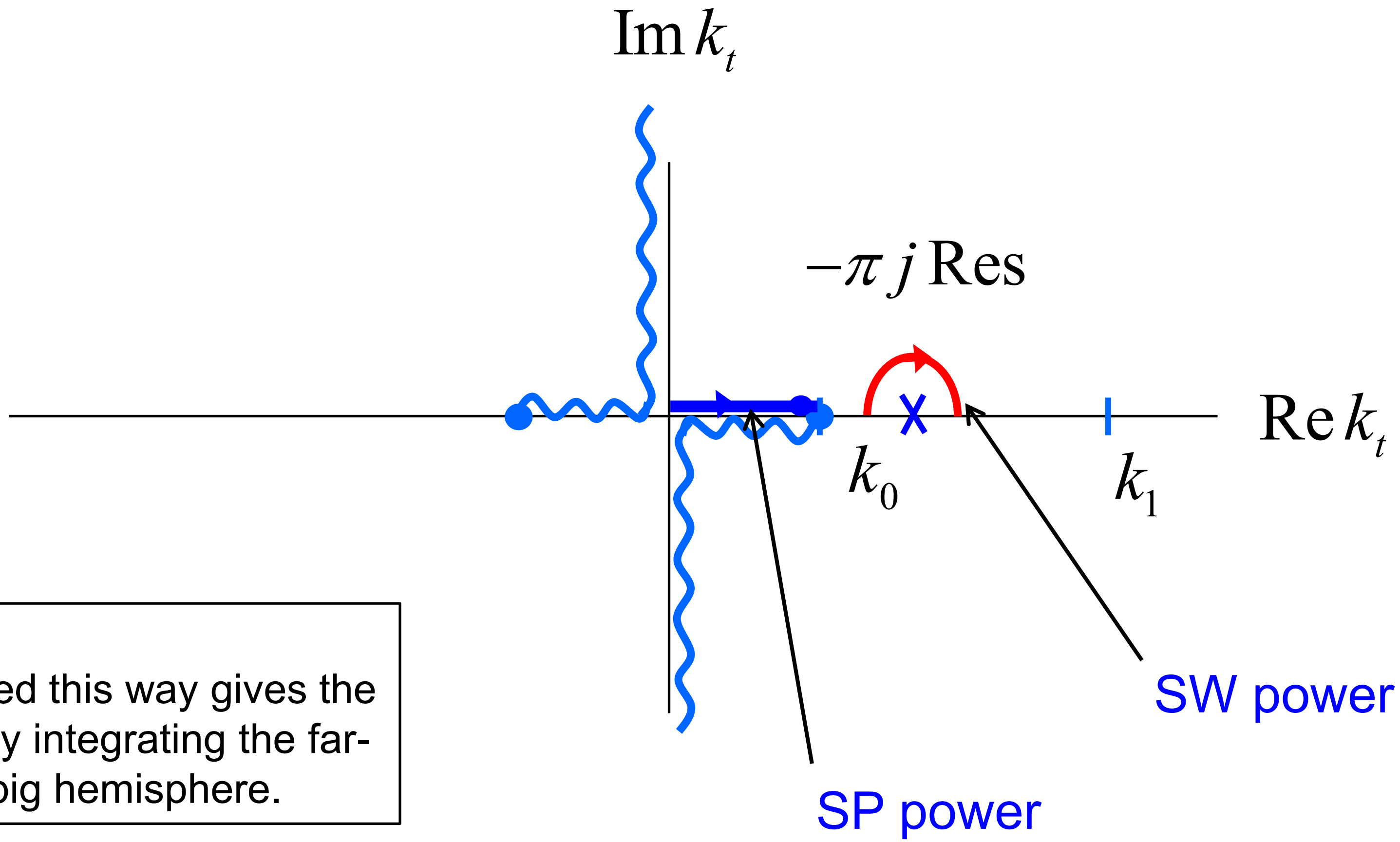
Hence, we have the following property:

$$\text{Re}\, \tilde{G}_{xx} = 0, \qquad k_t > k_0$$

$$D^{\text{TM}}(k_t) = \left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot\left(k_{z1}h\right)$$

Space-Wave and Surface-Wave Powers

The TM_0 pole gives a real-valued residue contribution:



Note:
The space-wave power calculated this way gives the same result that we would get by integrating the far-field power density over a big hemisphere.

Radiated Powers and Efficiency

$$P_{\mathrm{sp}} = \int_0^{\pi/2} \int_0^{k_0} \mathrm{Re}\, F_p\left(k_t, \bar{\phi}\right) \, dk_t d\bar{\phi}$$

$$P_{\mathrm{sw}} = \mathrm{Re} \int_0^{\pi/2} -j\pi \, \mathrm{Res}\, F_p\left(k_t, \bar{\phi}\right) d\bar{\phi}$$

$$e_r^{\mathrm{sw}} = \frac{P_{\mathrm{sp}}}{P_{\mathrm{sp}} + P_{\mathrm{sw}}}$$

Radiated Powers and Efficiency (cont.)

Alternatively,

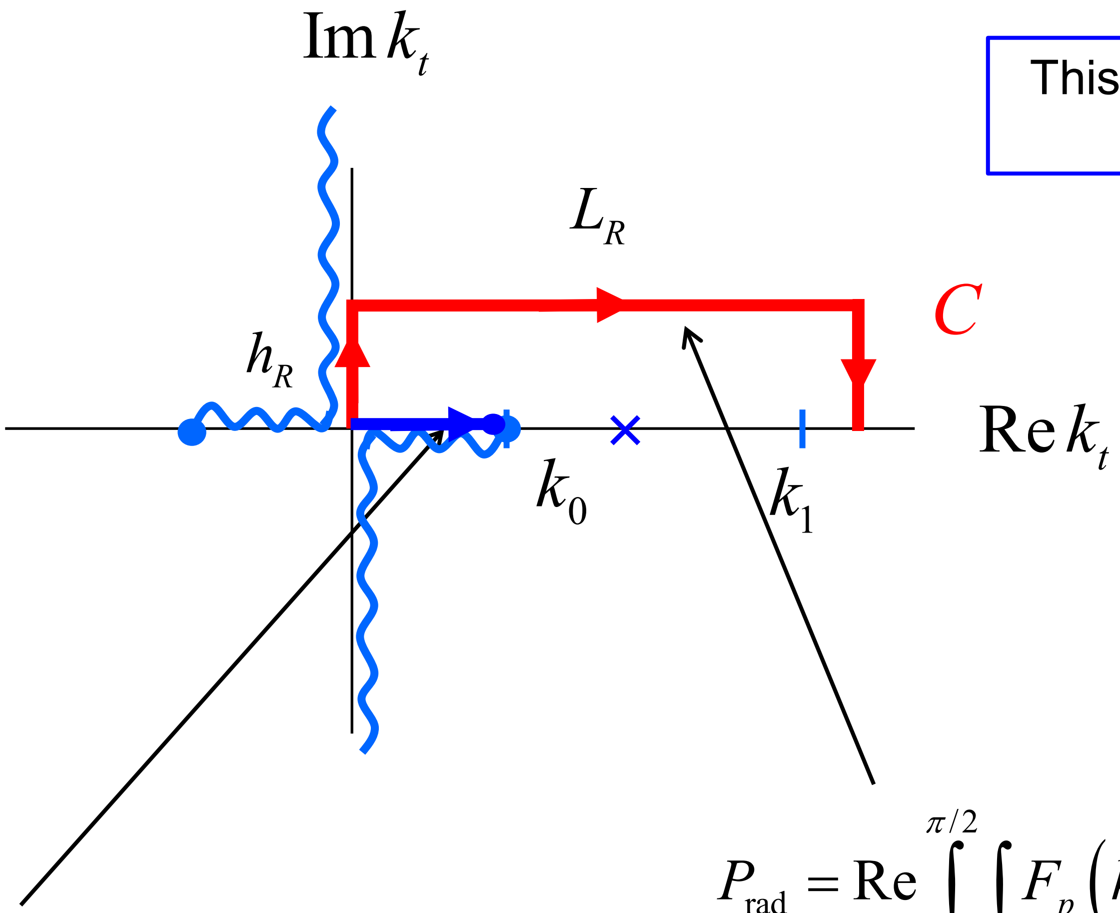
$$e_r^{\text{sw}} = \frac{P_{\text{sp}}}{P_{\text{rad}}}$$

$$P_{\text{sp}} = \int_0^{\pi/2}\int_0^{k_0} \operatorname{Re} F_p\left(k_t, \bar{\phi}\right)\, dk_t\, d\bar{\phi}$$

$$P_{\text{rad}} = \operatorname{Re}\int_0^{\pi/2}\int_C F_p\left(k_t, \bar{\phi}\right)\, dk_t\, d\bar{\phi}$$

The total radiated power (space + surface wave) comes from integrating along the rectangular path shown on the next slide.

Space-Wave Power and Total Radiated Power



This method does not require calculating the residue of the SW pole.

$$e_r^{\text{sw}} = \frac{P_{\text{sp}}}{P_{\text{rad}}}$$

$$P_{\text{rad}} = \operatorname{Re} \int_0^{\pi/2} \int_C F_p\left(k_t, \bar{\phi}\right) dk_t \, d\bar{\phi}$$

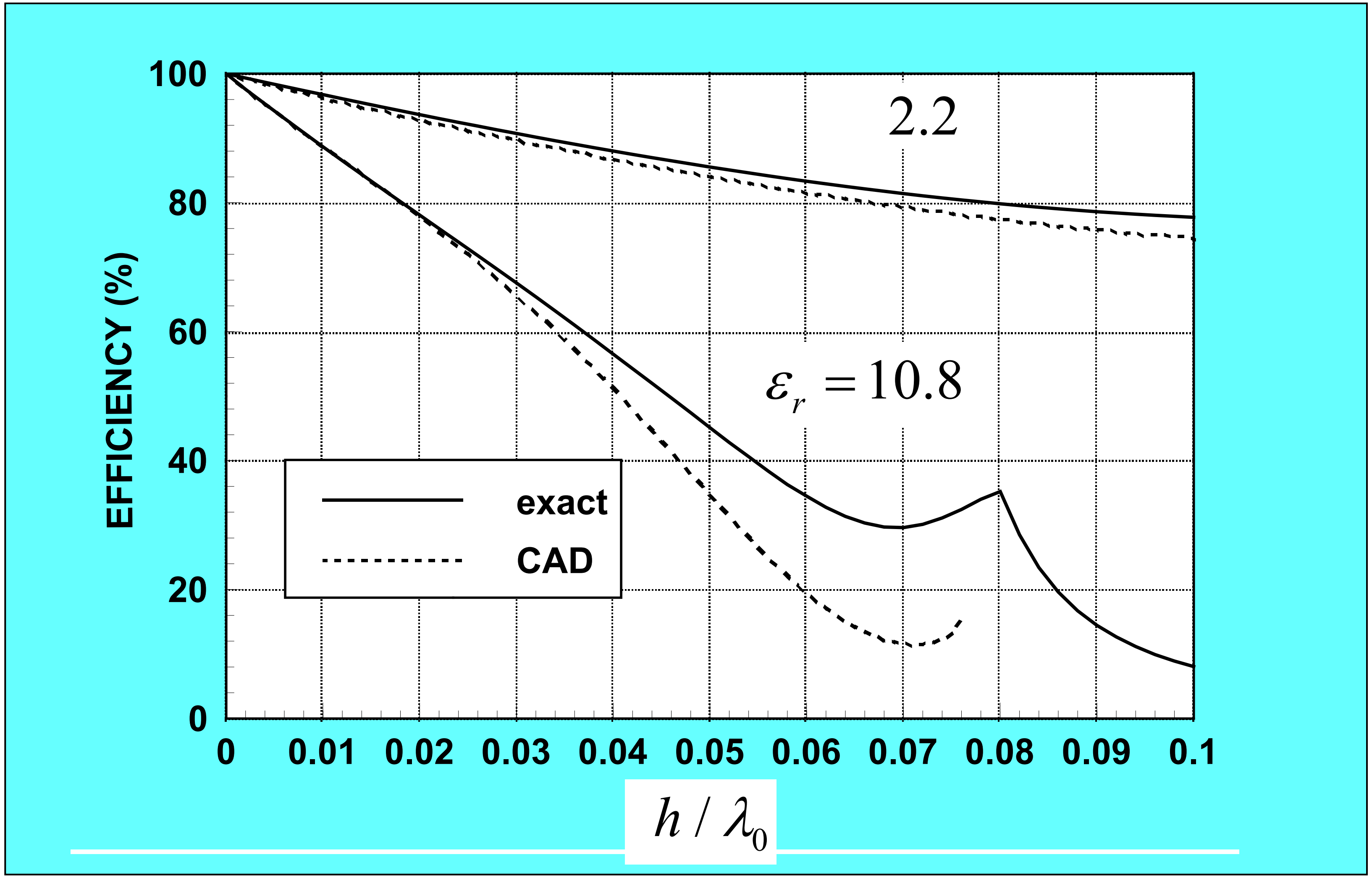
$$P_{\text{sp}} = \int_0^{\pi/2} \int_0^{k_0} \operatorname{Re} F_p\left(k_t, \bar{\phi}\right) dk_t \, d\bar{\phi}$$

Radiation Efficiency Reults

Results: Efficiency (Conductor and dielectric losses are neglected.)

$\varepsilon_r = 2.2$ or 10.8

$W / L = 1.5$



The loss of efficiency here is due only to the surface wave.