ECE 6345

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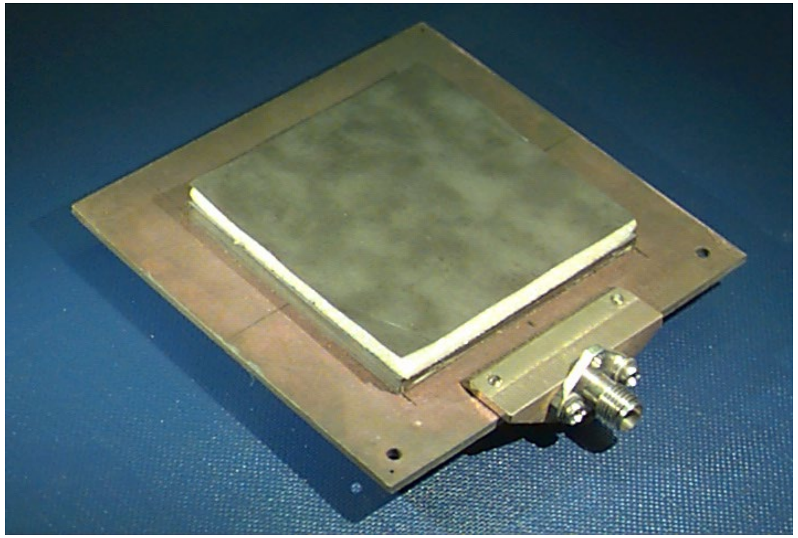

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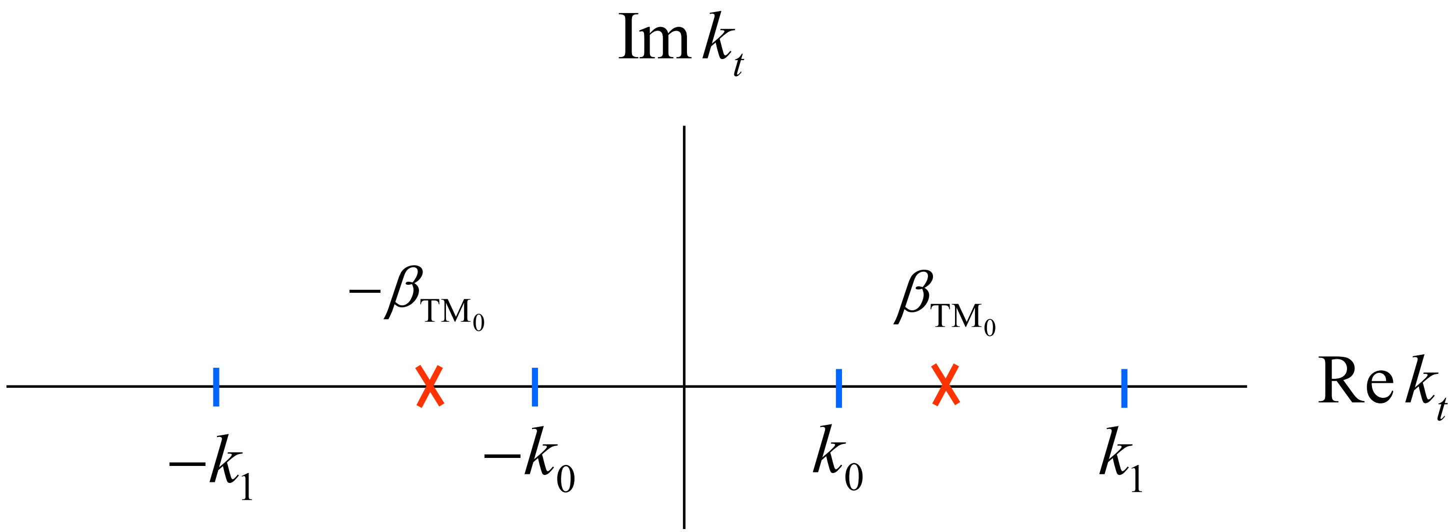
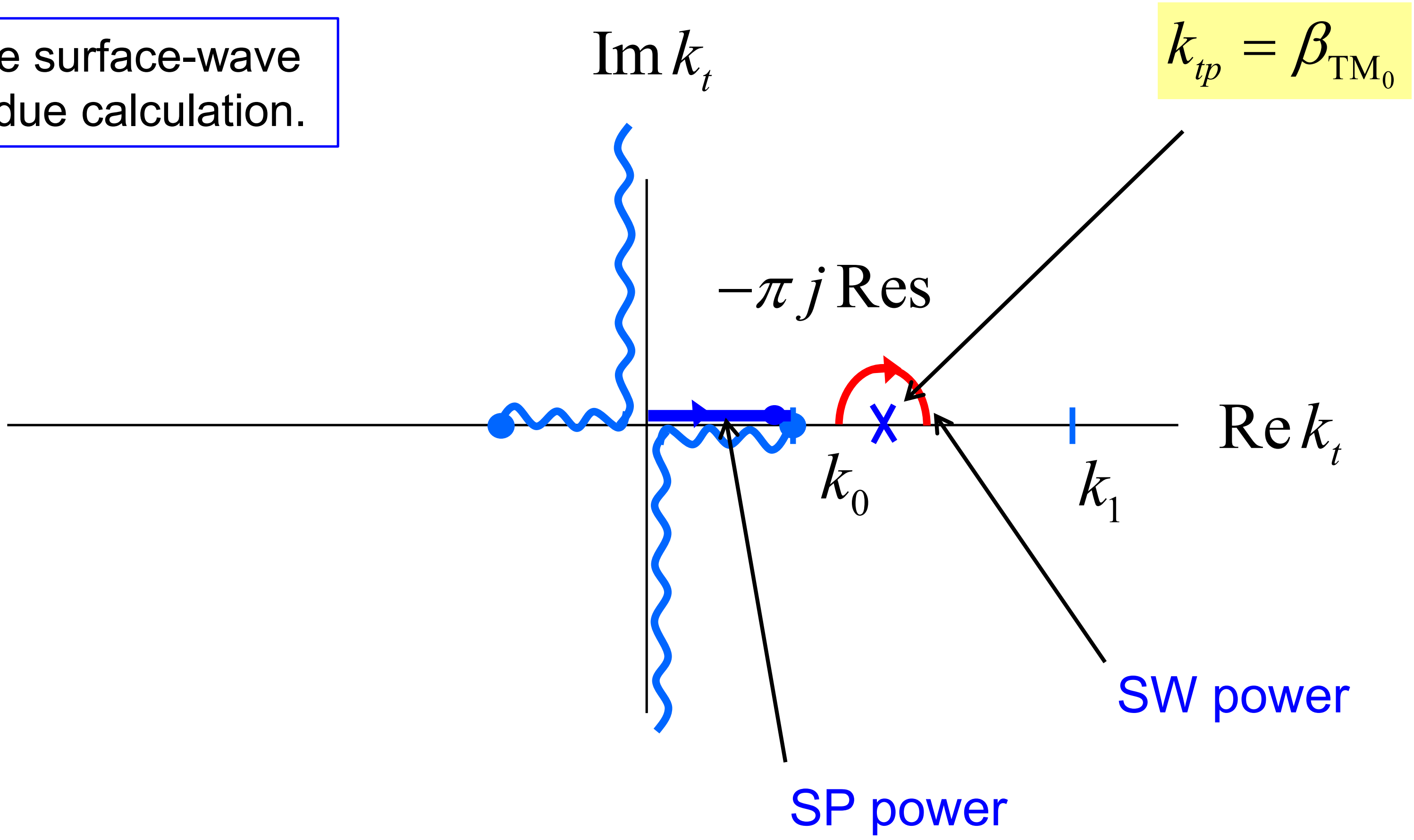
Overview

- In this set of notes we derive an approximate closed-form expression for the location of the TM_0 surface-wave pole, assuming a thin substrate.
- This is useful for later deriving a CAD formula for the surface-wave power, and from this, the surface-wave radiation efficiency, of a dipole source (next notes).



TM_0 Surface Wave Pole

Knowing the location of the surface-wave pole is important for a residue calculation.



CAD Formula for TM_0 Surface-Wave Pole

TRE:

$$\vec{Y}^{\mathrm{TM}} + \overleftarrow{Y}^{\mathrm{TM}} = 0$$

$$Y_0^{\mathrm{TM}} - jY_1^{\mathrm{TM}} \cot(k_{z1}h) = 0$$

$$\left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot(k_{z1}h) = 0$$

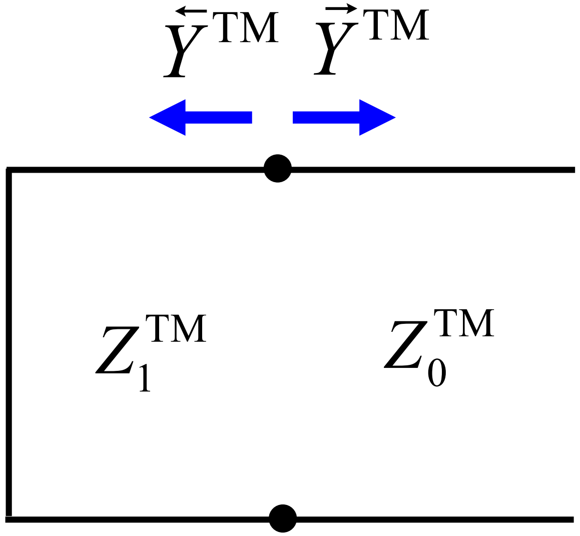
or

$$k_{z1} - j\varepsilon_r k_{z0} \cot\left(k_{z1}h\right) = 0$$

where

$$k_{z0} = \left(k_0^2 - \beta_{\mathrm{TM}_0}^2\right)^{1/2} = -j\alpha_{z0}$$

$$\alpha_{z0} = \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2}$$



Choose: $k_{z1} = \sqrt{k_1^2 - \beta_{\mathrm{TM}_0}^2}$

(The choice of square root is arbitrary for k_{z1}.)

CAD Formula for TM_0 Surface-Wave Pole (cont.)

$$k_{z1} - \alpha_{z0}\varepsilon_r \cot\left(k_{z1}h\right) = 0$$

or

$$\sqrt{k_1^2 - \beta_{\mathrm{TM}_0}^2} - \varepsilon_r \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2}\ \cot\left(h\sqrt{k_1^2 - \beta_{\mathrm{TM}_0}^2}\right) = 0$$

As $h \to 0$ we have:

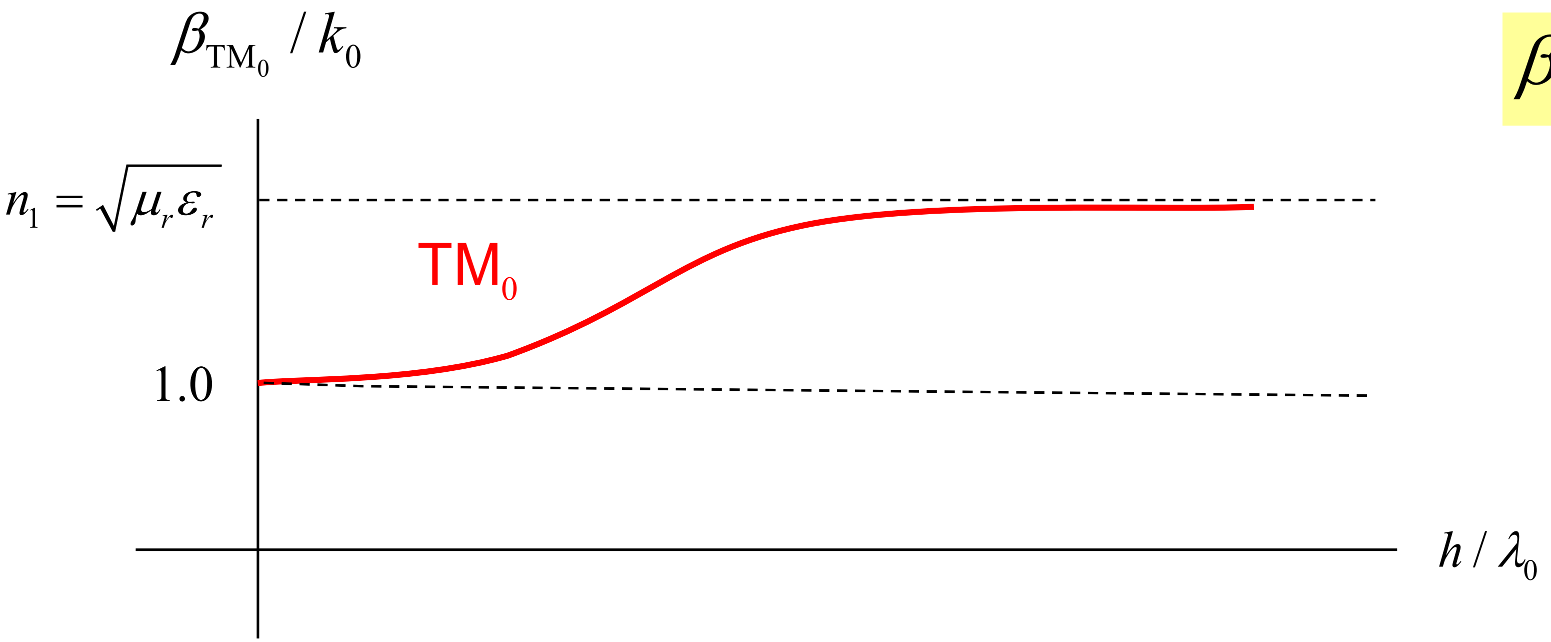
$$\sqrt{k_1^2 - \beta_{\mathrm{TM}_0}^2} = \varepsilon_r \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2}\left(\frac{1}{h\sqrt{k_1^2 - \beta_{\mathrm{TM}_0}^2}}\right) \Rightarrow \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2} = h\left(\frac{1}{\varepsilon_r}\left(k_1^2 - \beta_{\mathrm{TM}_0}^2\right)\right)$$

Hence $\beta_{\mathrm{TM}_0} \to k_0$ as $h \to 0$

CAD Formula for TM_0 Surface-Wave Pole (cont.)

$$k_{z1} - \alpha_{z0}\varepsilon_r \cot\left(k_{z1}h\right) = 0$$

Dispersion behavior of the TM_0 surface-wave mode.



$\beta_{\mathrm{TM}_0} \to k_0$ as $h / \lambda_0 \to 0$

CAD Formula for TM_0 Surface-Wave Pole (cont.)

To be more accurate for thin substrates, first re-write the TRE as:

$$k_1^2 - \beta_{\mathrm{TM}_0}^2 = \left(\frac{1}{h}\right)\varepsilon_r \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2}$$

or

$$k_1^2 - k_0^2 \approx \left(\frac{1}{h}\right)\varepsilon_r \sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2}$$

Recall:

$$\sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2} = h\left(\frac{1}{\varepsilon_r}\left(k_1^2 - \beta_{\mathrm{TM}_0}^2\right)\right)$$

Let $$\beta_{\mathrm{TM}_0}^2 = k_0^2\left(1+\Delta\right)$$

Then $$k_1^2 - k_0^2 \approx \left(\frac{1}{h}\right)\varepsilon_r k_0 \sqrt{\Delta}$$

CAD Formula for TM_0 Surface-Wave Pole (cont.)

We then have:

$$\Delta = \frac{h^2 \left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}$$

Hence, we have:

$$\beta_{\text{TM}_0} \approx k_0 \left[1 + \frac{h^2 \left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2} \right]^{1/2}$$

or

$$\beta_{\text{TM}_0} \approx k_0 \left[1 + \frac{\left(k_0 h\right)^2 \left(n_1^2 - 1\right)^2}{\varepsilon_r^2} \right]^{1/2}$$

Recall:

$$n_1 = \sqrt{\varepsilon_r \mu_r}$$