ECE 6345

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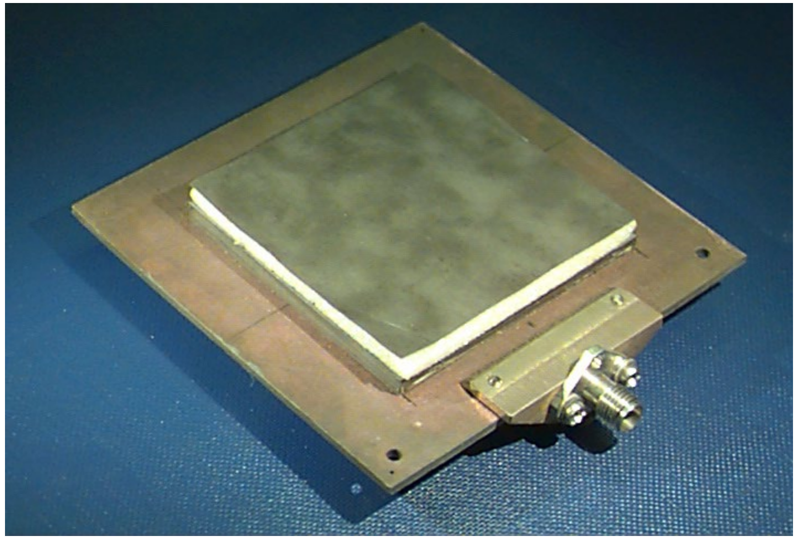
ECE Dept.



Notes 26

Overview

- In this set of notes we use the SDI method to calculate the surface-wave power radiated from an infinitesimal dipole (HED), and to obtain a CAD formula for it.

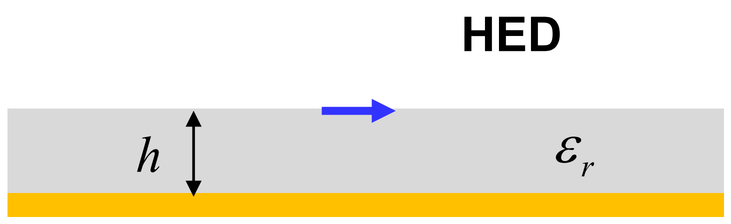
- We then obtain a CAD formula for the surface-wave radiation efficiency of the dipole. (This appears in the CAD formula for Q_{sw} of the patch.)

$$Q_{\mathrm{sw}} = Q_{\mathrm{sp}} \left(\frac{e_r^{\mathrm{sw}}}{1 - e_r^{\mathrm{sw}}} \right)$$

(Q_{sw} of Patch)

$$e_r^{\mathrm{sw}} \approx e_r^{\mathrm{hed}}$$

Approximation we make



Surface-Wave Power of Dipole

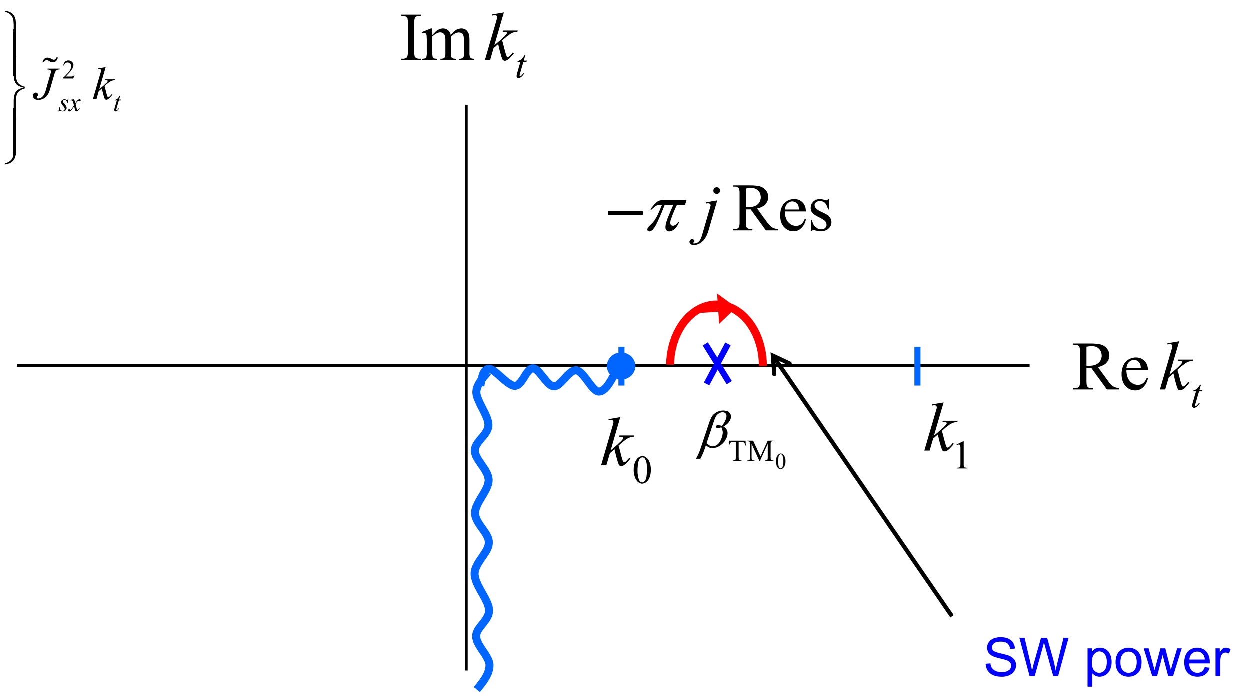
From Notes 24, we have the surface-wave power of a rectangular patch as

$$P_{sw} = \mathrm{Re} \int_0^{\pi/2} -\pi j \,\mathrm{Res}\, F_p\left(k_t, \bar{\phi}\right) d\bar{\phi}$$

$$F_p\left(k_t, \bar{\phi}\right) = \left(\frac{1}{2\pi^2}\right)\left\{\left(\frac{k_x}{k_t}\right)^2 V_i^{TM}(k_t) + \left(\frac{k_y}{k_t}\right)^2 V_i^{TE}(k_t)\right\} \tilde{J}_{sx}^2 \, k_t$$

$$V_i^{TM}(z) = \frac{1}{D^{TM}}$$

$$V_i^{TE}(z) = \frac{1}{D^{TE}}$$



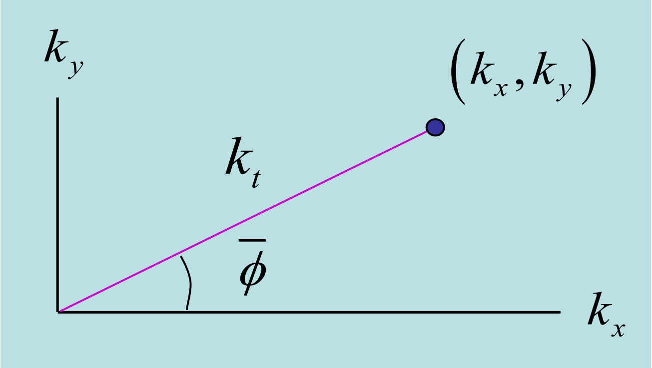
Surface-Wave Power of Dipole

$$F_p\left(k_t,\bar{\phi}\right)=\left(\frac{1}{2\pi^2}\right)\left\{\left(\frac{k_x}{k_t}\right)^2 V_i^{\mathrm{TM}}(k_t)\,\tilde{J}_{sx}^2+\left(\frac{k_y}{k_t}\right)^2 V_i^{\mathrm{TE}}(k_t)\,\tilde{J}_{sx}^2\right\}k_t$$

For a unit-amplitude infinitesimal dipole (HED) in the x direction we have:

$$J_{sx}(x,y)=\delta(x)\delta(y)$$

$$\tilde{J}_{sx}(k_x,k_y)=1$$



Hence, we have:

$$F_p\left(k_t,\bar{\phi}\right)=\frac{1}{2\pi^2}k_t\left\{\cos^2\bar{\phi}\;V_i^{\mathrm{TM}}(k_t)+\sin^2\bar{\phi}\;V_i^{\mathrm{TE}}(k_t)\right\}$$

Surface-Wave Power of Dipole (cont.)

At the TM_0 surface-wave pole, only the TM voltage function is infinite and has a residue. Therefore, the residue may be written as:

$$\operatorname{Res} F_p\left(k_t, \bar{\phi}\right) = \frac{1}{2\pi^2} \beta_{\mathrm{TM}_0} \cos^2 \bar{\phi} \;\; \operatorname{Res} V_i^{\mathrm{TM}}\left(\beta_{\mathrm{TM}_0}\right)$$

Hence, integrating in the spectral angle variable, the surface-wave power becomes:

$$P_{\mathrm{sw}} = \frac{\pi}{4} \operatorname{Re}\left\{ -\pi j \frac{1}{2\pi^2} \beta_{\mathrm{TM}_0} \operatorname{Res} V_i^{\mathrm{TM}}\left(\beta_{\mathrm{TM}_0}\right) \right\}$$

Note: $\displaystyle\int_0^{\pi/2} \cos^2 \bar{\phi} \, d\bar{\phi} = \frac{\pi}{4}$

Surface-Wave Power of Dipole (cont.)

We thus have:

$$P_{\text{sw}} = \frac{-1}{8}\beta_{\text{TM}_0}\,\text{Re}\left\{ j\,\text{Res}\,V_i^{\text{TM}}\left(\beta_{\text{TM}_0}\right)\right\}$$

Recall that $V_i^{\text{TM}}\left(k_t\right) = \dfrac{1}{D^{\text{TM}}(k_t)}$

We then have that $$\text{Res}\;V_i^{\text{TM}}\left(\beta_{\text{TM}_0}\right) = \frac{1}{\left.\dfrac{dD^{\text{TM}}}{dk_t}\right|_{k_t=\beta_{\text{TM}_0}}}$$

Next, recall that

$$D^{\text{TM}}\left(k_t\right) = \left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot\left(k_{z1}h\right)$$

Surface-Wave Power of Dipole (cont.)

$$D^{\mathrm{TM}}(k_t) = \left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot(k_{z1}h)$$

Taking the derivative, we have:

$$\frac{dD^{\mathrm{TM}}}{dk_t} = (\omega\varepsilon_0)\left(-\frac{1}{k_{z0}^2}\right)\frac{dk_{z0}}{dk_t} - (j\omega\varepsilon_1)\left(-\frac{1}{k_{z1}^2}\right)\frac{dk_{z1}}{dk_t}\cot(k_{z1}h)$$
$$-(j\omega\varepsilon_1)\left(\frac{1}{k_{z1}}\right)\left(-\csc^2(k_{z1}h)\right)h\frac{dk_{z1}}{dk_t}$$

where

$$k_{z0} = \left(k_0^2 - k_t^2\right)^{1/2} \qquad \frac{dk_{z0}}{dk_t} = \frac{1}{2}\left(k_0^2 - k_t^2\right)^{-1/2}(-2k_t) = \frac{-k_t}{k_{z0}}$$

$$k_{z1} = \left(k_1^2 - k_t^2\right)^{1/2} \qquad \frac{dk_{z1}}{dk_t} = \frac{1}{2}\left(k_1^2 - k_t^2\right)^{-1/2}(-2k_t) = \frac{-k_t}{k_{z1}}$$

CAD Formula for Surface-Wave Power

To simplify, assume a thin substrate: $h \to 0: \;\; \beta_{\mathrm{TM}_0} \to k_0, \; k_{z0} \to 0$

Examine the behavior of the three terms in the previous result:

$$\text{Term 1} \propto \frac{1}{k_{z0}^3}$$

$$\text{Term 2} \propto \frac{1}{h}$$

$$\text{Term 3} \propto \frac{1}{h}$$

From Notes 25:

$$\beta_{\mathrm{TM}_0}^2 = k_0^2 \left(1 + \Delta\right)$$

where

$$\Delta = \frac{h^2 \left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}$$

$$\Rightarrow \quad k_{z0} = -j\sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2} \approx -jk_0\sqrt{\Delta} \propto h$$

CAD Formula for Surface-Wave Power (cont.)

Hence, keeping only Term1:

$$\frac{dD^{\mathrm{TM}}}{dk_t} = \left(\omega\varepsilon_0\right)\left(-\frac{1}{k_{z0}^2}\right)\frac{dk_{z0}}{dk_t} \approx \left(\omega\varepsilon_0\right)\left(\frac{k_t}{k_{z0}^3}\right)$$

We then have:

$$\operatorname{Res} V_i^{\mathrm{TM}}\left(\beta_{\mathrm{TM}_0}\right) = \frac{1}{\left.\dfrac{dD^{\mathrm{TM}}}{dk_t}\right|_{k_t=\beta_{\mathrm{TM}_0}}} \approx \frac{k_{z0}^3}{\left(\omega\varepsilon_0\right)\beta_{\mathrm{TM}_0}}$$

$$\approx \frac{\left(-jk_0\sqrt{\Delta}\right)^3}{\left(\omega\varepsilon_0\right)\beta_{\mathrm{TM}_0}}$$

$$\approx \left(\frac{jk_0^3}{\left(\omega\varepsilon_0\right)\beta_{\mathrm{TM}_0}}\right)\Delta^{3/2}$$

$$= \left(\frac{jk_0^3}{\left(\omega\varepsilon_0\right)\beta_{\mathrm{TM}_0}}\right)\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3 k_0^3}$$

Recall:

$$k_{z0} = -j\sqrt{\beta_{\mathrm{TM}_0}^2 - k_0^2} \approx -jk_0\sqrt{\Delta}$$

$$\Delta = \frac{h^2\left(k_1^2-k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}$$

CAD Formula for Surface-Wave Power (cont.)

Hence, we have:

$$P_{sw} \sim -\frac{1}{8}\beta_{TM_0}\left(\frac{-k_0^3}{(\omega\varepsilon_0)\beta_{TM_0}}\right)\left(\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3 k_0^3}\right)$$

$$=\frac{1}{8}\left(\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3(\omega\varepsilon_0)}\right)$$

$$=\frac{1}{8}\left(\frac{(k_0h)^3\left(n_1^2-1\right)^3}{\varepsilon_r^3(\omega\varepsilon_0)}\right)k_0^3$$

Recall:

$$P_{sw}=\frac{-1}{8}\beta_{TM_0}\,\mathrm{Re}\left\{j\,\mathrm{Res}\,V_i^{TM}\left(\beta_{TM_0}\right)\right\}$$

$$\mathrm{Res}\;V_i^{TM}\left(\beta_{TM_0}\right)\approx\left(\frac{jk_0^3}{(\omega\varepsilon_0)\beta_{TM_0}}\right)\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3 k_0^3}$$

Next, use $\omega\varepsilon_0=\dfrac{k_0}{\eta_0}$

CAD Formula for Surface-Wave Power (cont.)

We then have:

$$P_{\text{sw}} = \frac{\eta_0}{8}(k_0 h)^3 k_0^2 \left(\frac{\left(n_1^2 - 1\right)}{\varepsilon_r} \right)^3 = \frac{\eta_0}{8}(k_0 h)^3 k_0^2 \left(n_1^2 \frac{\left(1 - \frac{1}{n_1^2}\right)}{\varepsilon_r} \right)^3$$

$$\left(\frac{n_1^2}{\varepsilon_r} = \mu_r \right)$$

or

$$P_{\text{sw}} = \frac{\eta_0}{8}(k_0 h)^3 k_0^2 \mu_r^3 \left(1 - \frac{1}{n_1^2} \right)^3$$

CAD Formula for Radiation Efficiency

Recall that for a unit-amplitude dipole ($Il = 1$) on a thin substrate, we have (Notes 12):

$$P_{\text{sp}} \approx (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

We can now calculate the radiation efficiency of the dipole:

$$e_r^{\text{hed}} = \frac{P_{\text{sp}}}{P_{\text{sp}} + P_{\text{sw}}} = \frac{1}{1 + \left(\frac{P_{\text{sw}}}{P_{\text{sp}}} \right)}$$

CAD Formula for Radiation Efficiency (cont.)

Substituting in for the powers, we have:

$$e_r^{\text{hed}} = \frac{1}{1+\left(\dfrac{P_{\text{sw}}}{P_{\text{sp}}}\right)}$$

$$= \frac{1}{1+\left(\dfrac{\dfrac{\eta_0}{8}(k_0 h)^3 k_0^2 \mu_r^3 \left(1-\dfrac{1}{n_1^2}\right)^3}{(k_0 h)^2 k_0^2 \left(\dfrac{\eta_0}{6\pi}\right)\mu_r^2 c_1}\right)}$$

For a unit-amplitude HED:

$$P_{\text{sw}} = \frac{\eta_0}{8}(k_0 h)^3 k_0^2 \mu_r^3 \left(1-\frac{1}{n_1^2}\right)^3$$

$$P_{\text{sp}} \approx (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi}\right)\mu_r^2 c_1$$

CAD Formula for Radiation Efficiency (cont.)

$$e_r^{\text{hed}} = \frac{1}{1 + \left(\dfrac{\dfrac{\eta_0}{8}\left(k_0 h\right)^3 k_0^2 \mu_r^3 \left(1 - \dfrac{1}{n_1^2}\right)^3}{\left(k_0 h\right)^2 k_0^2 \left(\dfrac{\eta_0}{6\pi}\right) \mu_r^2 c_1} \right)}$$

Simplifying, we have:

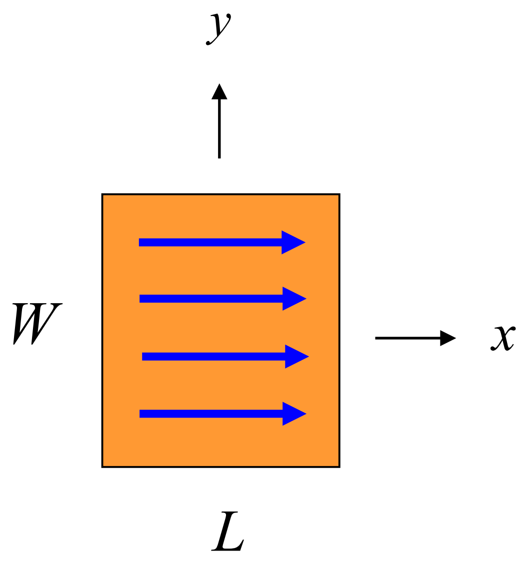
$$e_r^{\text{hed}} \approx \frac{1}{1 + \left(\dfrac{3\pi}{4}\right)\left(\dfrac{1}{c_1}\right)\mu_r \left(1 - \dfrac{1}{n_1^2}\right)^3 \left(k_0 h\right)}$$

This is the last remaining CAD formula!

CAD Formula for Q_{sw}

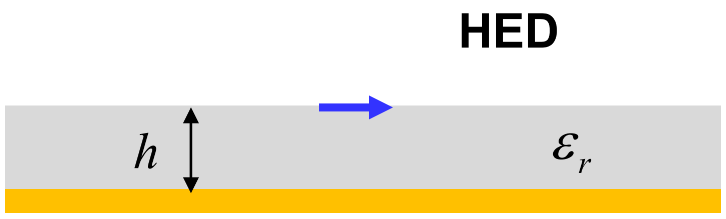
Patch

$$Q_{\mathrm{sw}} = Q_{\mathrm{sp}}\left(\frac{e_r^{\mathrm{sw}}}{1-e_r^{\mathrm{sw}}}\right)$$



HED

$$e_r^{\mathrm{sw}} \approx e_r^{\mathrm{hed}}$$



$$e_r^{\mathrm{hed}} \approx \frac{1}{1+\left(\frac{3\pi}{4}\right)\left(\frac{1}{c_1}\right)\mu_r\left(1-\frac{1}{n_1^2}\right)^3 (k_0 h)}$$