ECE 6345

Spring 2024

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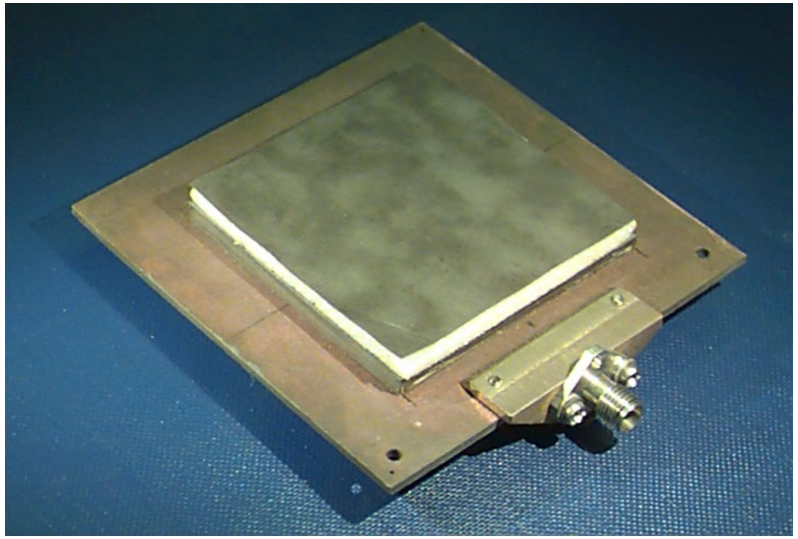
ECE Dept.

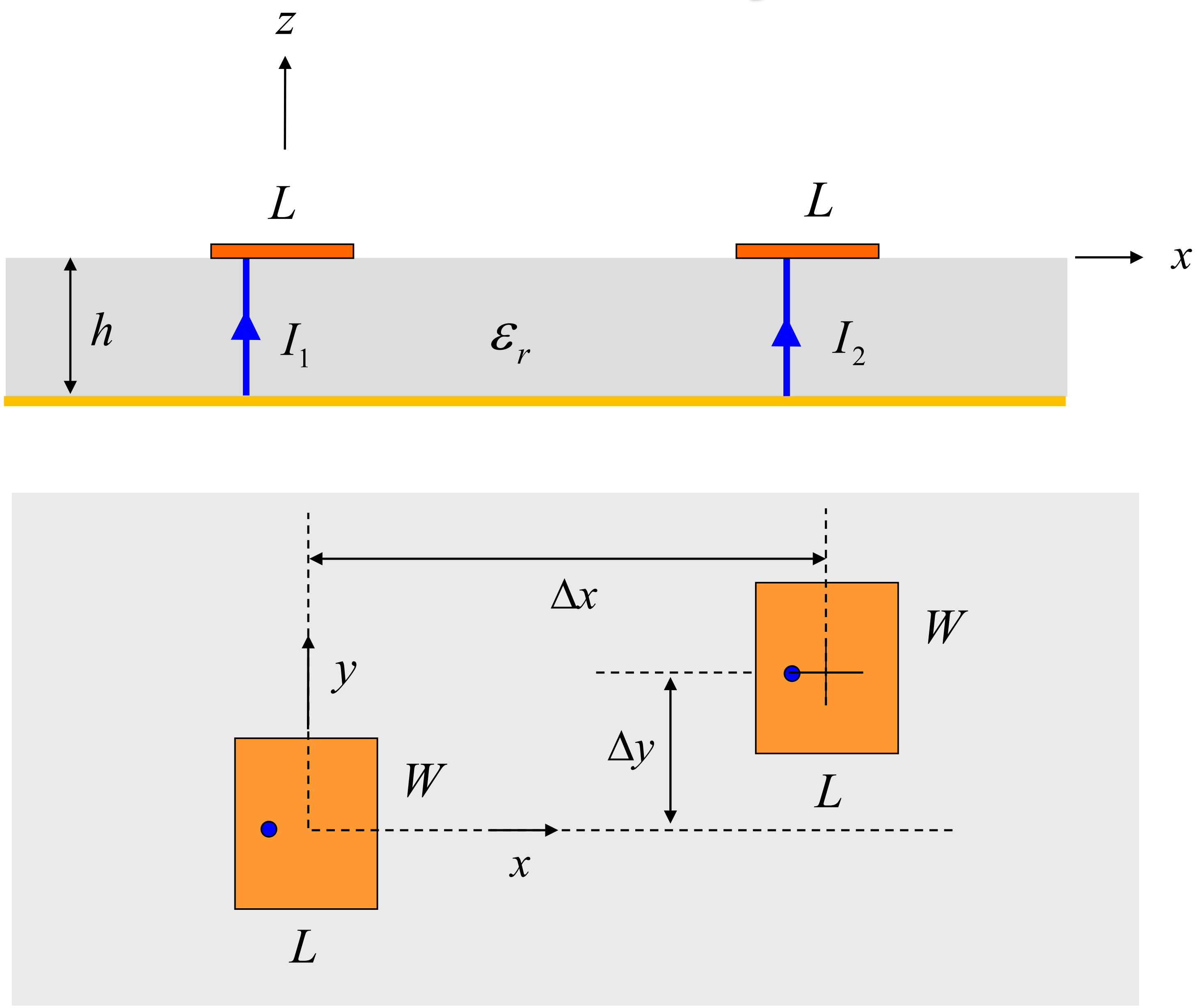


Notes 27

Overview

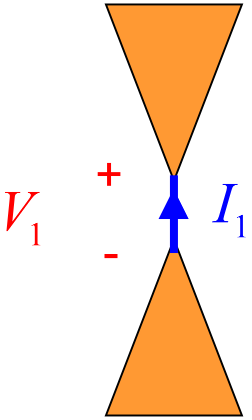
- In this set of notes we use the spectral-domain method to find the mutual impedance between two rectangular patch antennas.

Geometry

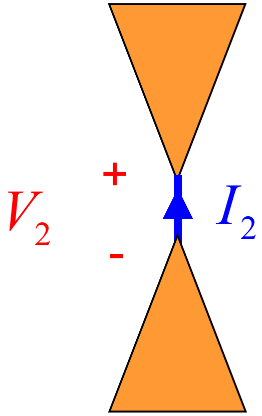


Mutual Impedance Formulation

Assume two arbitrary antennas, to be general.



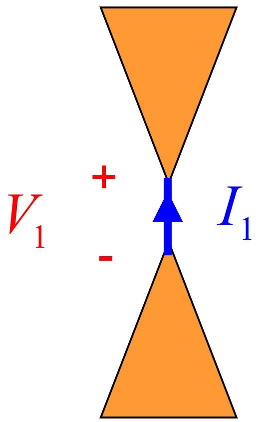
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



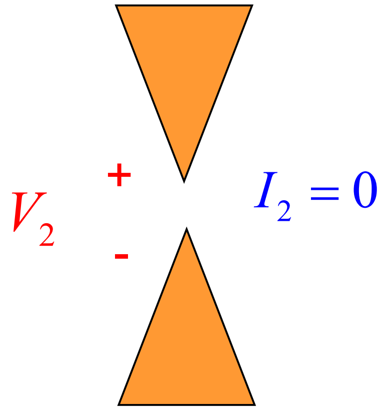
The two-port system is described by a 2×2 impedance (Z) matrix.

Mutual Impedance Formulation (cont.)

The self impedance Z_{11} is calculated.



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{11} \approx Z_{\text{in}}$$

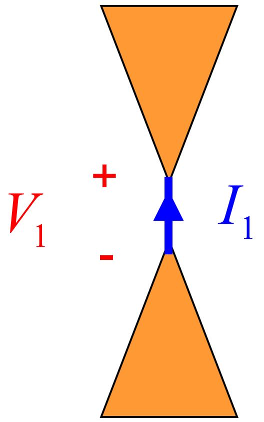
(The presence of open-circuited antenna 2 does not significantly affect the input impedance of antenna 1.)

Mutual Impedance Formulation (cont.)

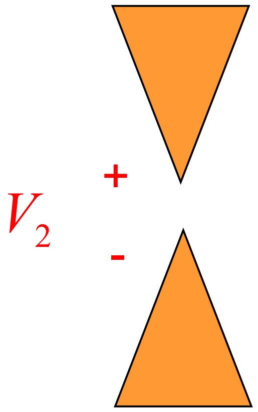
The mutual impedance Z_{21} is calculated.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$

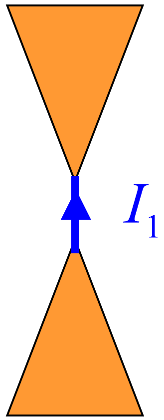


Note: $Z_{21} = Z_{12}$

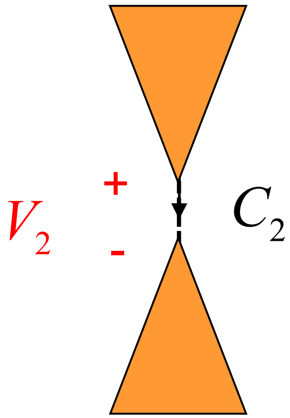
Mutual Impedance Formulation (cont.)

The open-circuit voltage V_2 is obtained by integrating the electric field produced from current I_1 over the path C_2.

$$V_2 = \int_{C_2} \underline{E}_1 \cdot d\underline{r}$$

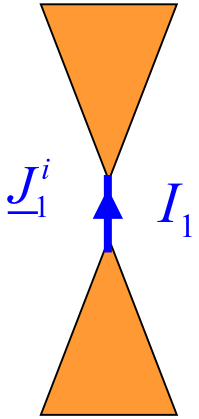


$\underline{E}_1$ = electric field produced by the feed current I_1, in the presence of antenna 1 and antenna 2.



Mutual Impedance Formulation (cont.)

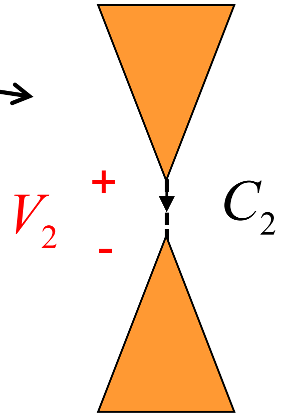
The open-circuit voltage V_2 is put in the form of a reaction.

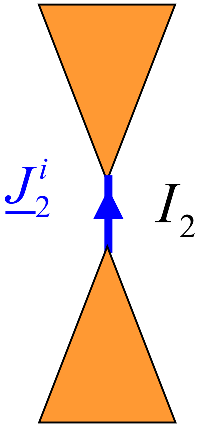


$$V_2 = \int_{C_2} \underline{E}_1 \cdot d\underline{r}$$

$$= -\frac{1}{I_2} \int_V \left(\underline{E}_1 \cdot \underline{J}_2^i \right) dV$$

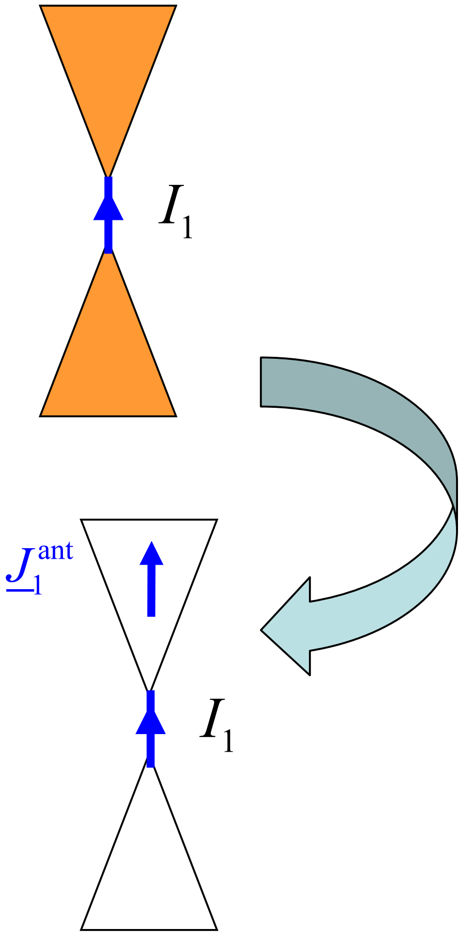
$$= -\frac{1}{I_2} \left\langle \underline{J}_1^i , \underline{J}_2^i \right\rangle$$





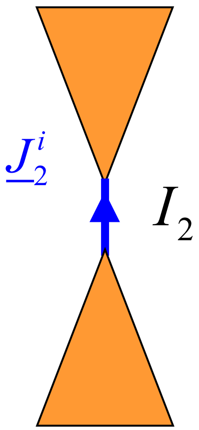
Mutual Impedance Formulation (cont.)

The equivalence principle is used to replace antenna 1 with its surface current.



$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^i, \underline{J}_2^i \right\rangle$$

$$= -\frac{1}{I_2}\left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^i \right\rangle$$

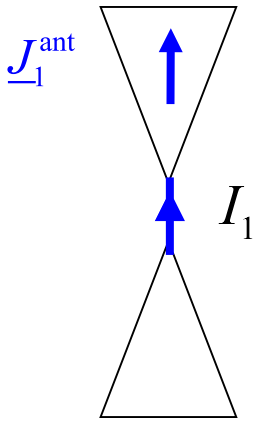


The field produced by current I_1 exciting antenna 1 is the same as that produced by the current on antenna 1 in free space.

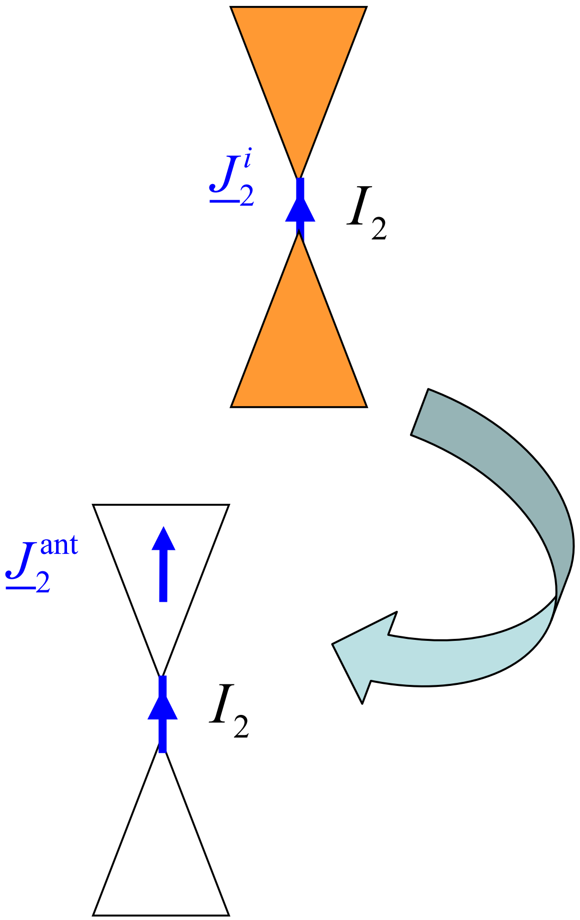
The antenna current $\underline{J}_1^{\text{ant}}$ is that excited on antenna 1 when it is in the <u>presence</u> of open-circuited antenna 2.

Mutual Impedance Formulation (cont.)

Reciprocity is invoked, and then the equivalence principle.



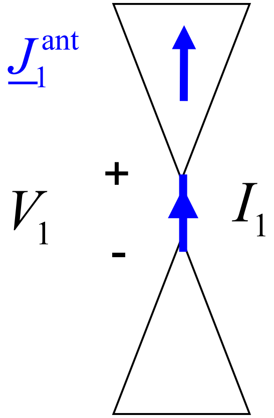
$$
\begin{aligned}
V_2 &= -\frac{1}{I_2}\left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^{i} \right\rangle \\
&= -\frac{1}{I_2}\left\langle \underline{J}_2^{i}, \underline{J}_1^{\text{ant}} \right\rangle \\
&= -\frac{1}{I_2}\left\langle \underline{J}_2^{\text{ant}}, \underline{J}_1^{\text{ant}} \right\rangle
\end{aligned}
$$



The antenna current $\underline{J}_2^{\text{ant}}$ is that excited on antenna 2 when it is in the <u>absence</u> of open-circuited antenna 1.

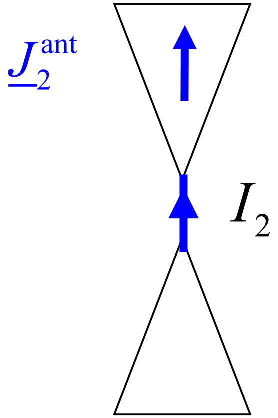
Mutual Impedance Formulation (cont.)

Reciprocity is invoked one more time.



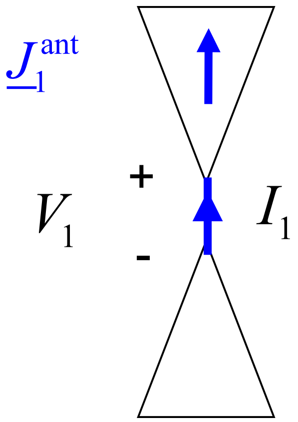
$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_2^{\text{ant}}, \underline{J}_1^{\text{ant}} \right\rangle$$

$$= -\frac{1}{I_2}\left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^{\text{ant}} \right\rangle$$

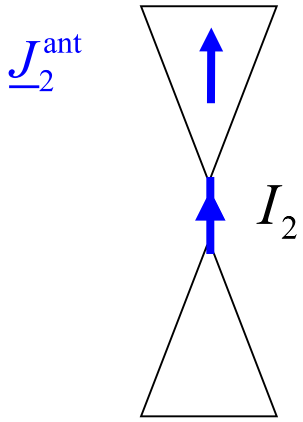


Mutual Impedance Formulation (cont.)

$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^{\text{ant}} \right\rangle$$



$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

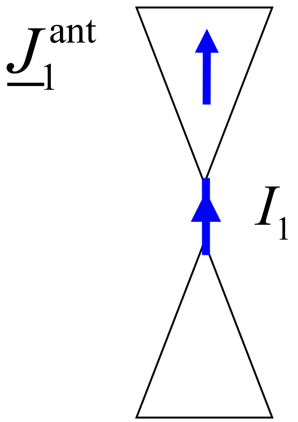


The mutual impedance is then:

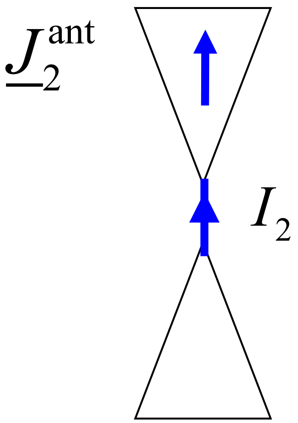
$$Z_{21} = -\frac{1}{I_1 I_2}\left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^{\text{ant}} \right\rangle$$

Mutual Impedance Formulation (cont.)

Summary



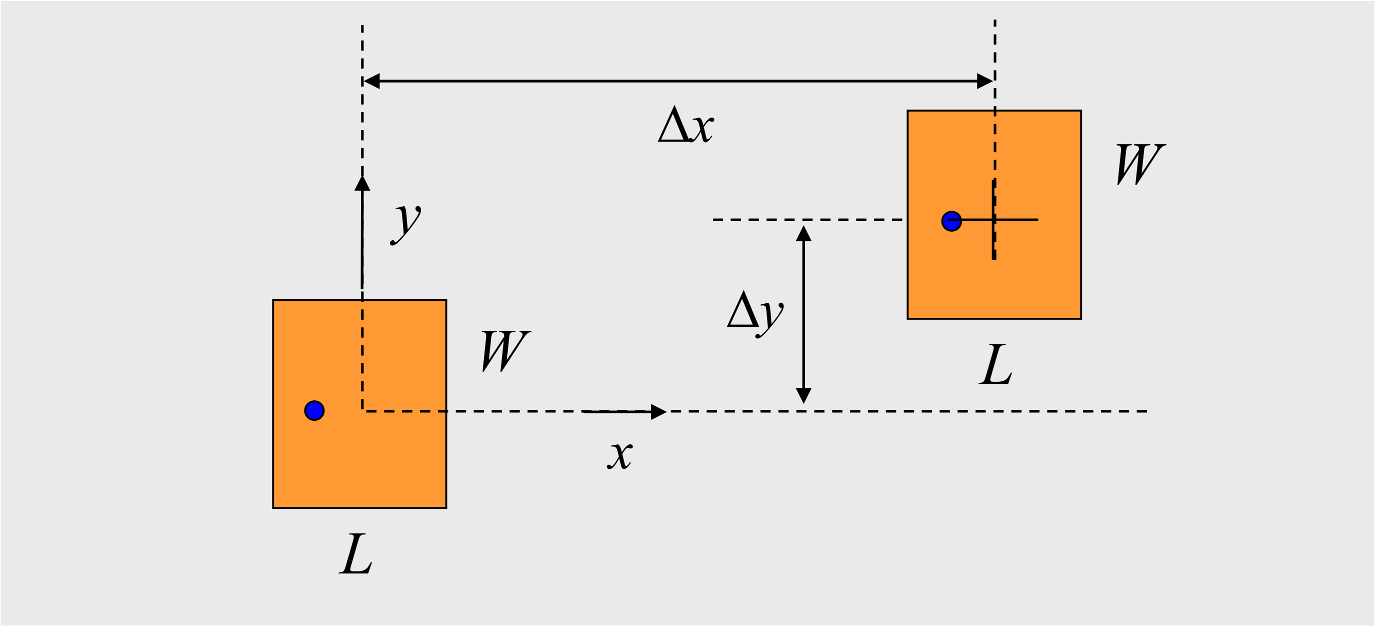
$$Z_{12} = -\frac{1}{I_1 I_2} \left\langle \underline{J}_1^{\text{ant}}, \underline{J}_2^{\text{ant}} \right\rangle$$



$J_1^{\text{ant}} =$ current on antenna 1, when excited by current I_1 in the presence of open-circuited antenna 2.

$J_2^{\text{ant}} =$ current on antenna 2, when excited by current I_2 in the absence of antenna 1.

Mutual Impedance Between two Patch Antennas



$$Z_{12} = -\frac{1}{I_1 I_2} \left\langle J_{sx}^{(1)}, J_{sx}^{(2)} \right\rangle$$

The two patches are assumed to be identical here.

Assume $I_1 = I_2 = 1\ [\mathrm{A}]$

Mutual Impedance Between Patches (cont.)

$$Z_{12} = -\left\langle J_{sx}^{(1)}, J_{sx}^{(2)} \right\rangle = -A_x^{(1)} A_x^{(2)} \left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -A_x^2 \left\langle B_x^{(1)}, B_x^{(2)} \right\rangle$$

Note : $A_x^{(1)} = A_x^{(2)} = A_x$

Denote $Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle$

A_x is the amplitude of the patch current when the patch is fed by a 1 A probe current.

Then we have $Z_{12} = A_x^2 Z_{xx}^{1,2}$

$$B_x^{(1)} = \cos\left(\frac{\pi x}{L}\right)$$

$$B_x^{(2)} = \cos\left(\frac{\pi\left(x-\Delta x\right)}{L}\right)$$

"basis functions"

From Notes 9:

$$\frac{A_{10}^J}{I_0} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{\text{in}}}{h\sin\left(\frac{\pi x_0}{L}\right)}\right] \Rightarrow A_x = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{\text{in}}}{h\sin\left(\frac{\pi x_0}{L}\right)}\right]$$

New notation: $\left(A_x = A_{10}^J, \;\; I_0 = 1 \text{ A}\right)$

Mutual Impedance Between Patches (cont.)

Calculation of reaction $Z_{xx}^{1,2}$ between patch basis functions:

$$\tilde{E}_x\left[B_x^{(1)}\right] = \tilde{G}_{xx}\tilde{B}_x^{(1)}$$

$$E_x\left[B_x^{(1)}\right] = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{G}_{xx}\tilde{B}_x^{(1)}\,e^{-j\left(k_x x + k_y y\right)}\,dk_x dk_y$$

Hence, integrating over the surface of patch 2, we have:

$$Z_{xx}^{1,2} = -\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{G}_{xx}\left(k_x,k_y\right)\,\tilde{B}_x^{(1)}\left(k_x,k_y\right)\tilde{B}_x^{(2)}\left(-k_x,-k_y\right)\,dk_x dk_y$$

$$\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}B_{x2}\left(x,y\right)e^{-j\left(k_x x + k_y y\right)}\,dxdy = \tilde{B}_2\left(-k_x,-k_y\right)\right)$$

Mutual Impedance Between Patches (cont.)

$$Z_{xx}^{1,2} = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}(k_x, k_y)\, \tilde{B}_x^{(1)}(k_x, k_y)\, \tilde{B}_x^{(2)}(-k_x, -k_y)\, dk_x dk_y$$

From the Fourier “shifting” theorem, we have:

$$\tilde{B}_x^{(2)}(k_x, k_y) = \tilde{B}_x^{(1)}(k_x, k_y)\, e^{j(k_x \Delta x + k_y \Delta y)} \quad \Rightarrow \quad \tilde{B}_x^{(2)}(-k_x, -k_y) = \tilde{B}_x^{(1)}(-k_x, -k_y)\, e^{-j(k_x \Delta x + k_y \Delta y)}$$

Hence, we have:

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}(k_x, k_y) \left(\tilde{B}_x^{(1)}(k_x, k_y) \right)^2 \; e^{-j(k_x \Delta x + k_y \Delta y)} dk_x dk_y$$

Note: $\tilde{B}_x^{(1)}(k_x, k_y) = \left(\frac{\pi}{2} LW\right) \operatorname{sinc}\left(k_y \frac{W}{2}\right) \left[\frac{\cos\left(k_x \frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(k_x \frac{L}{2}\right)^2} \right]$ $\quad \left(\tilde{B}_x^{(1)}(-k_x, -k_y) = \tilde{B}_x^{(1)}(k_x, k_y) \right)$

Mutual Impedance Between Patches (cont.)

Converting to polar coordinates, we have:

Note: The "1" superscript is dropped henceforth.

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) e^{-j\left(k_x \Delta x + k_y \Delta y\right)} k_t \, dk_t \, d\overline{\phi}$$

Since the integrand is an even function of k_x and k_y, we can write:

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_0^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\overline{\phi}$$

Quadrant 1 | Quadrant 2 | Quadrant 3 | Quadrant 4

Note:
$$e^{-j\left(k_x \Delta x\right)} e^{-j\left(k_y \Delta y\right)} + e^{+j\left(k_x \Delta x\right)} e^{-j\left(k_y \Delta y\right)} + e^{+j\left(k_x \Delta x\right)} e^{+j\left(k_y \Delta y\right)} + e^{-j\left(k_x \Delta x\right)} e^{+j\left(k_y \Delta y\right)}$$

$$= 2\cos\left(k_x \Delta x\right) e^{-j\left(k_y \Delta y\right)} + 2\cos\left(k_x \Delta x\right) e^{+j\left(k_y \Delta y\right)}$$

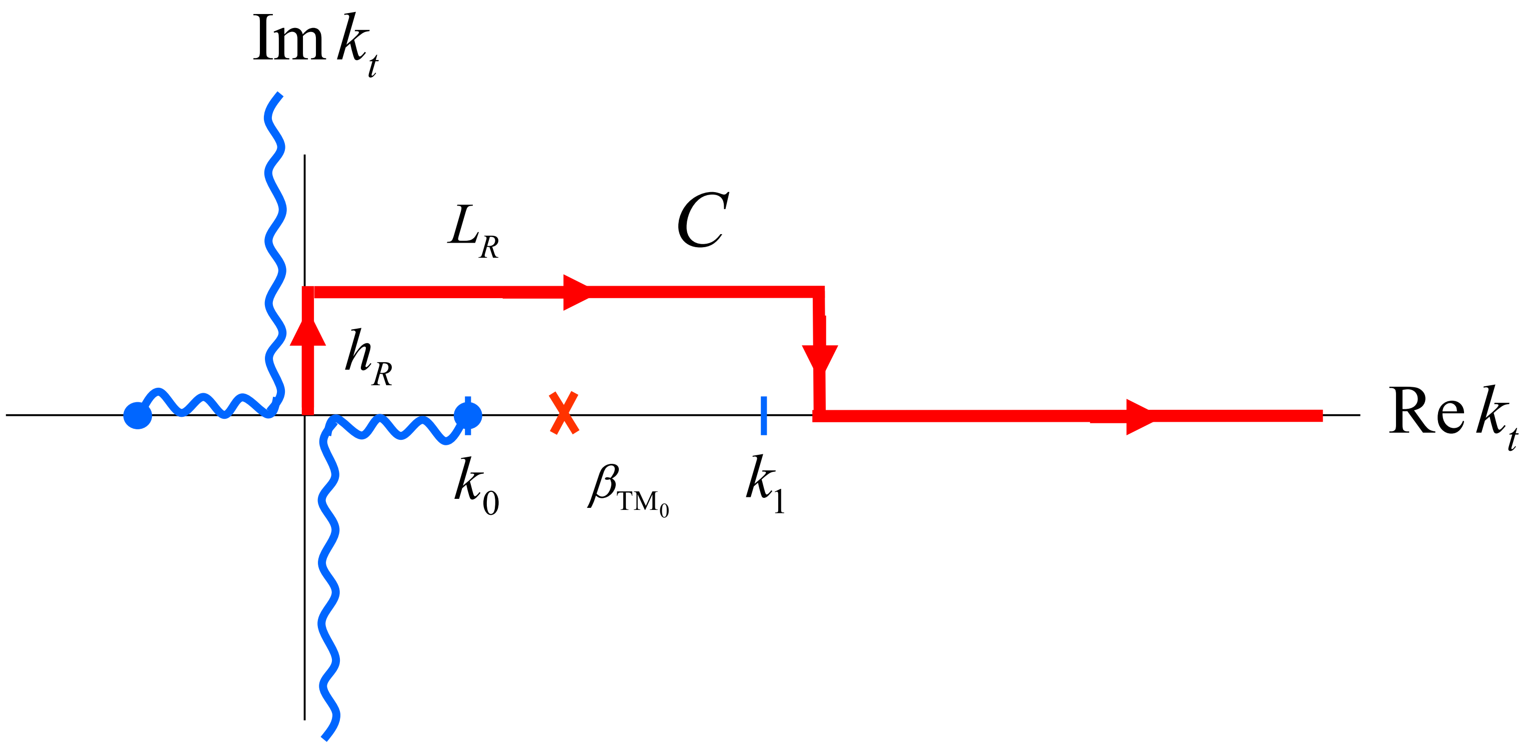
$$= 2\cos\left(k_x \Delta x\right)\left[e^{-j\left(k_y \Delta y\right)} + e^{+j\left(k_y \Delta y\right)}\right]$$

$$= 4\cos\left(k_x \Delta x\right)\cos\left(k_y \Delta y\right)$$

Mutual Impedance Between Patches (cont.)

Final form of mutual reaction:

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\overline{\phi}$$



Summary

$$Z_{12} = A_x^2 \, Z_{xx}^{1,2}$$

$$Z_{xx}^{1,2} = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \, \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) \, k_t \, dk_t \, d\overline{\phi}$$

$$A_x = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{\text{in}}}{h \sin\left(\frac{\pi x_0}{L}\right)}\right]$$

Results

D. M. Pozar, "Input Impedance and mutual coupling of rectangular microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30. pp. 1191-1196, Nov. 1982.

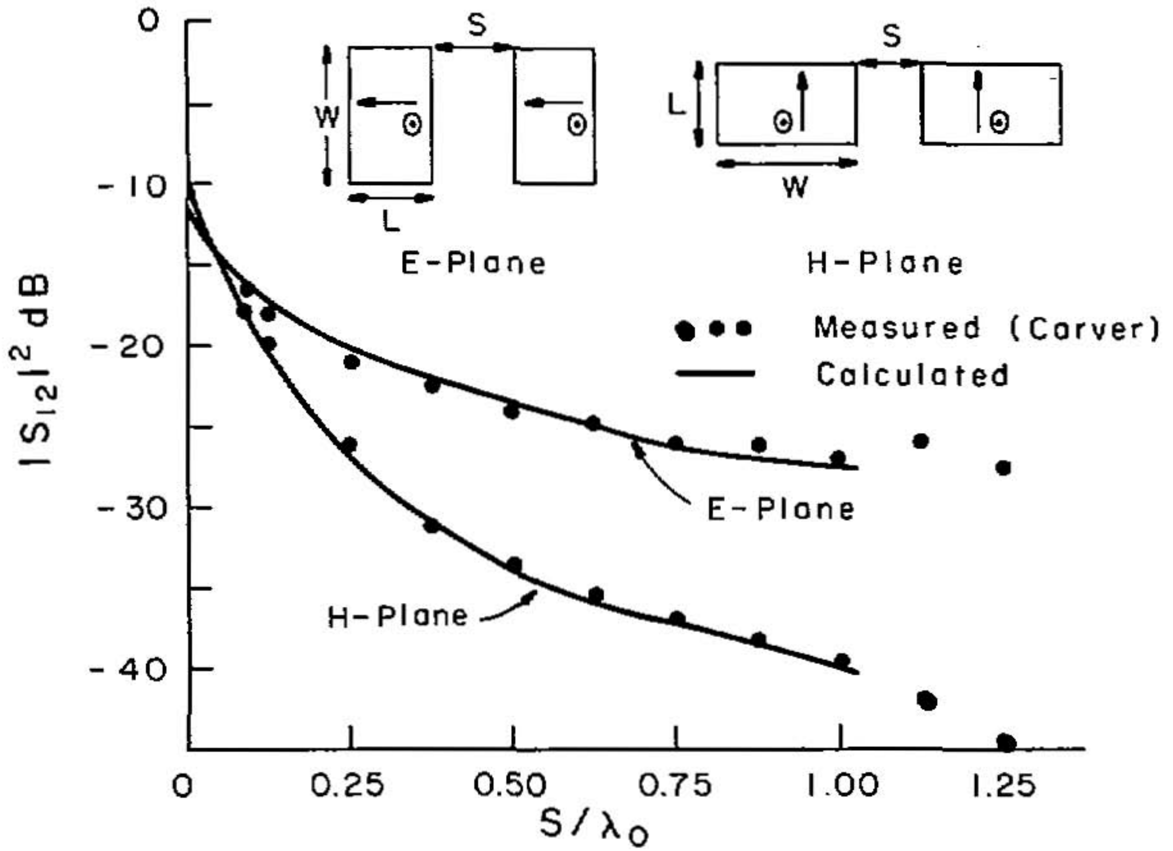


Fig. 8. Measured and calculated mutual coupling between two coax-fed microstrip antennas, for both $\bar{E}$-plane and $\bar{H}$-plane coupling. W = 10.57 cm, L = 6.55 cm, d = 0.1588 cm, ϵ_r = 2.55, f = 1410. MHz.

Components of Mutual Impedance

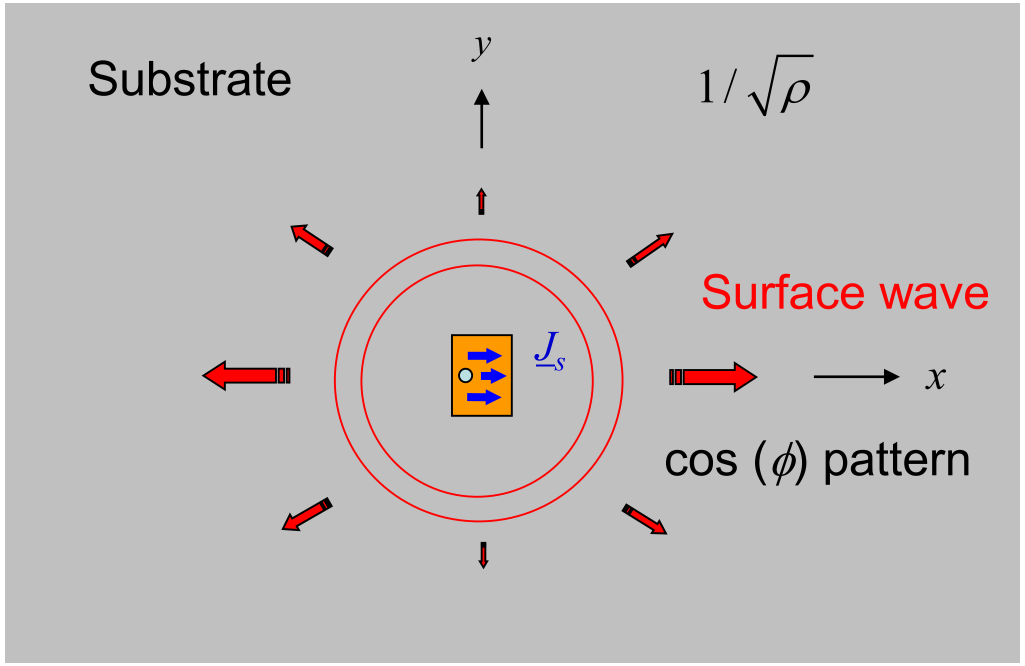
We can also consider the various components of Z_{12}:

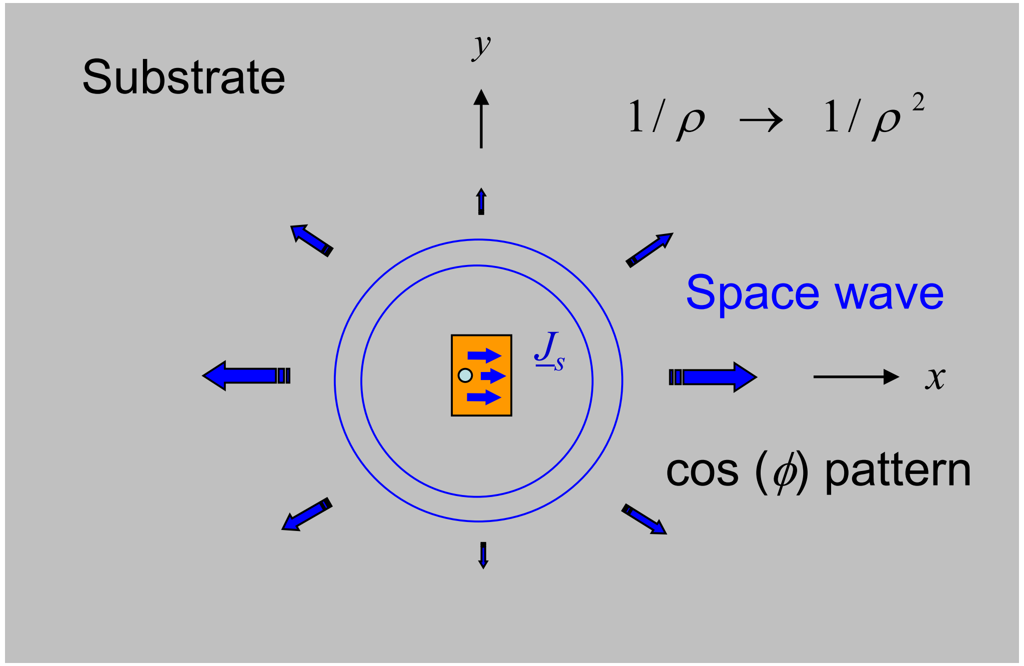
- Surface-wave contribution
- Space-wave contribution ("lateral wave")

❖ The space wave decays as $1/\sqrt{\rho}$

❖ The space wave initially decays as $1/\rho$

(but eventually transitions to a behavior of $1/\rho^2$)





Components of Mutual Impedance (cont.)

Results for typical patches

Circular patches

$$\varepsilon_r = 2.94, \quad h / \lambda_0 = 0.01, \quad f = 2.0\,\text{GHz} \qquad (h = 1.5\,\text{mm})$$