ECE 6345

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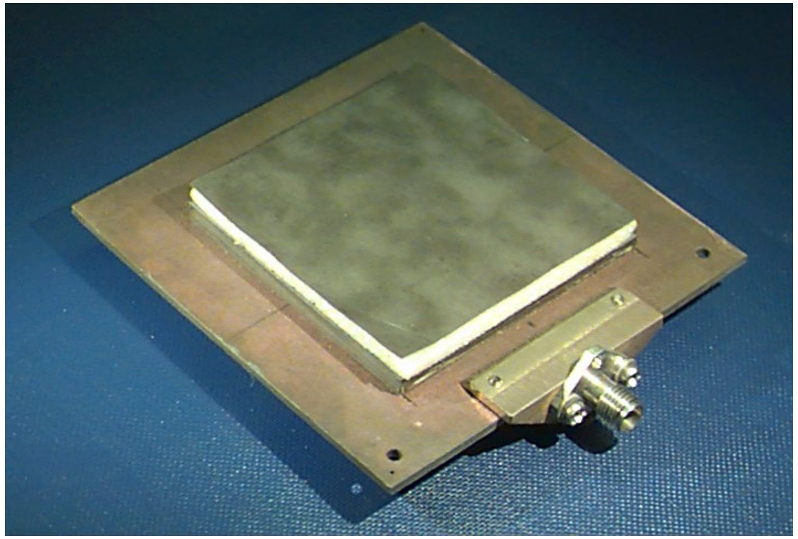
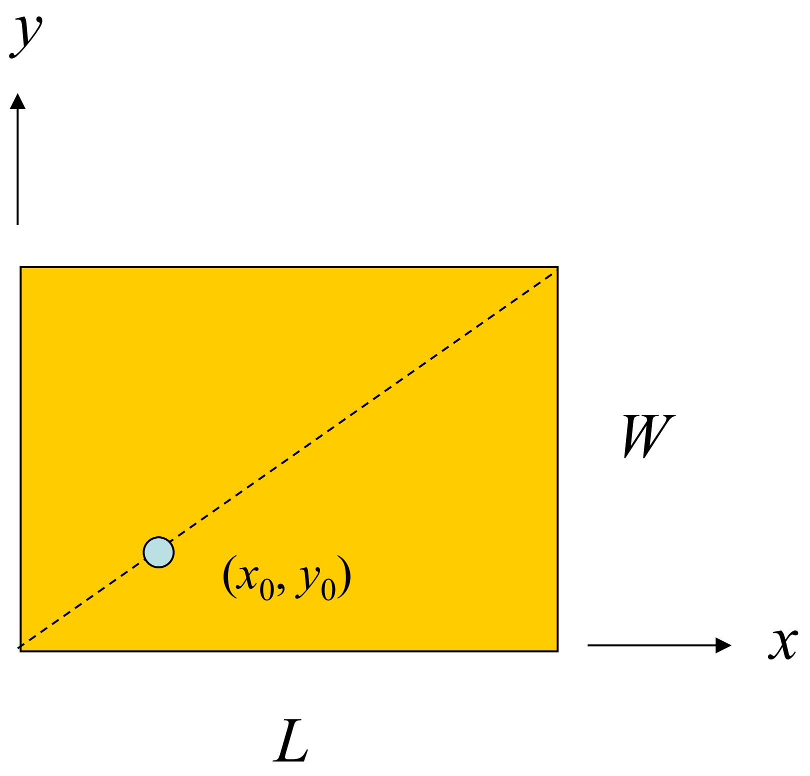


Notes 6

Overview

This set of notes treats circular polarization, obtained by using a single feed.

The nearly-square patch is fed with a coaxial feed along the diagonal.



$$L \approx W$$

$$y_0 \approx x_0$$

Overview

Goals:

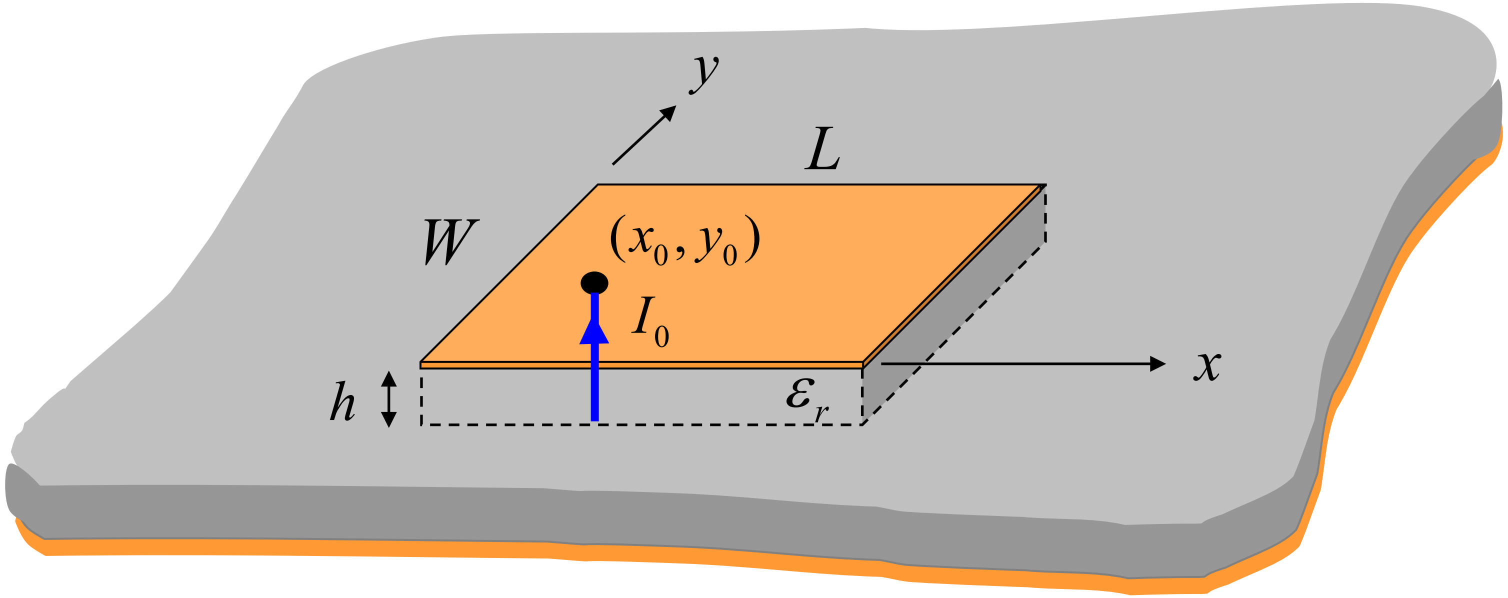
- Find the optimum dimensions of the CP patch
- Find the input impedance of the CP patch
- Find the pattern (axial-ratio) bandwidth of the CP patch
- Find the impedance bandwidth of the CP patch

We use the electric current model, and thus work with the currents on the patch.

Amplitude of Patch Currents

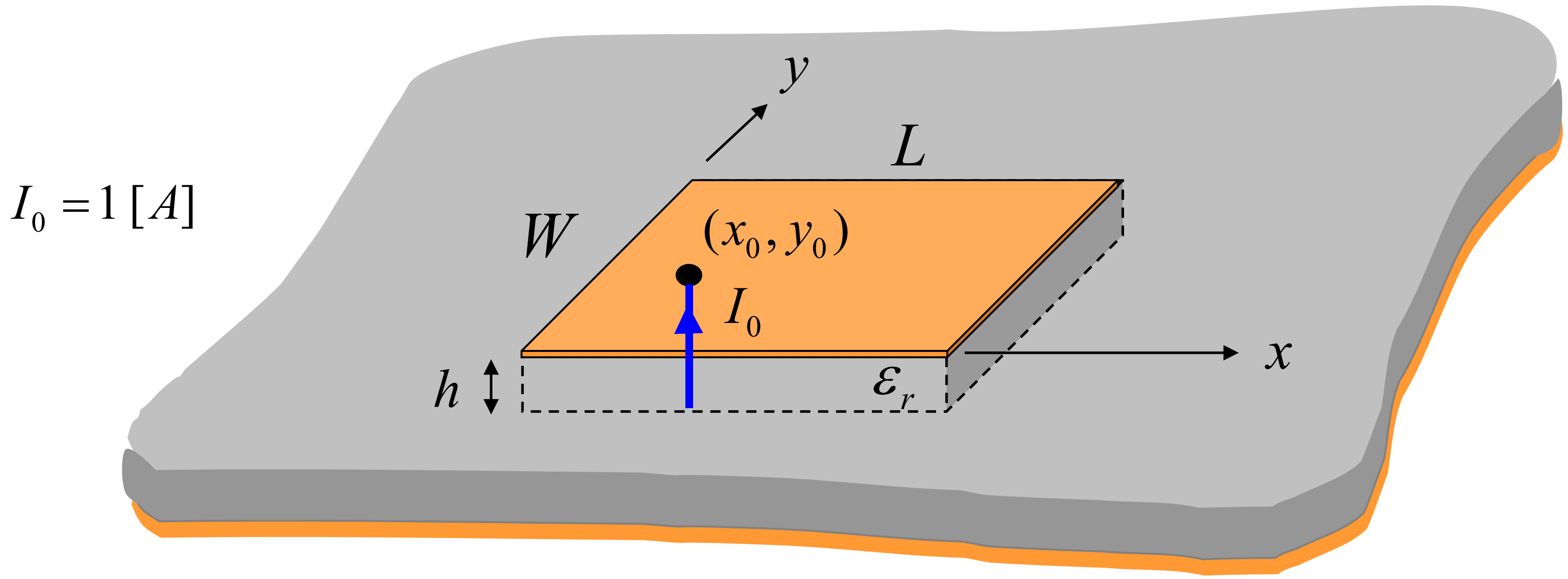
First Step:

Find the patch currents (x and y directions), and then relate them to the input impedance of the patch.



Assume: $I_0 = 1\ [A]$

Amplitude of Patch Currents (cont.)



Patch currents:

x-directed current mode (TM_{10}): $\quad \underline{J}_s^x = \underline{\hat{x}}\, A_x \sin\left(\dfrac{\pi x}{L}\right)$

y-directed current mode (TM_{01}): $\quad \underline{J}_s^y = \underline{\hat{y}}\, A_y \sin\left(\dfrac{\pi y}{W}\right)$

Amplitude of Patch Currents (cont.)

The x mode (TM_{10}):

$$\underline{J}_s = \underline{\hat{n}} \times \underline{H} = -\underline{\hat{z}} \times \underline{H}$$

so $J_{sx} = H_y$

Assume: $\underline{H} = \underline{\hat{y}}\, A_x \sin\left(\frac{\pi x}{L}\right)$

To find $\underline{E}$, use $\nabla \times \underline{H} = j\omega\varepsilon \underline{E}$

$$E_z = \frac{1}{j\varepsilon_0\varepsilon_r}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = \frac{1}{j\varepsilon_0\varepsilon_r} A_x \left(\frac{\pi}{L}\right)\cos\left(\frac{\pi x}{L}\right)$$

Amplitude of Patch Currents (cont.)

$$Z_{\text{in}} = \frac{V}{I_0} = V = -hE_z = -h\left(E_z^x + E_z^y\right) = Z_{\text{in}}^x + Z_{\text{in}}^y$$

For the TM_{10} mode we have, from the last slide:

$$Z_{\text{in}}^x = \frac{jh}{\omega\varepsilon_0\varepsilon_r}\left(\frac{\pi}{L}\right)A_x \cos\left(\frac{\pi x_0}{L}\right)$$

And thus, solving for A_x:

$$A_x = \frac{Z_{\text{in}}^x}{\cos\left(\frac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$

A similar derivation holds for the y mode (TM_{01}).

Amplitude of Patch Currents (cont.)

The y mode (TM_{01}):

$$A_y = \frac{Z_{\text{in}}^y}{\cos\left(\dfrac{\pi y_0}{W}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r W}{j\pi h}\right]$$

The patch current amplitudes can then be written as:

$$A_x = A_1^x Z_{\text{in}}^x$$

$$A_y = A_1^y Z_{\text{in}}^y$$

where

$$A_1^x = \frac{1}{\cos\left(\dfrac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$

$$A_1^y = \frac{1}{\cos\left(\dfrac{\pi y_0}{W}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r W}{j\pi h}\right]$$

$$\underline{J}_s^x = \underline{\hat{x}}\, A_x \sin\left(\frac{\pi x}{L}\right)$$

$$\underline{J}_s^y = \underline{\hat{y}}\, A_y \sin\left(\frac{\pi y}{W}\right)$$

Amplitude of Patch Currents (cont.)

$$A_1^x = \frac{1}{\cos\left(\frac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right] \qquad A_1^y = \frac{1}{\cos\left(\frac{\pi y_0}{W}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r W}{j\pi h}\right]$$

Assume:

$$L \approx W$$

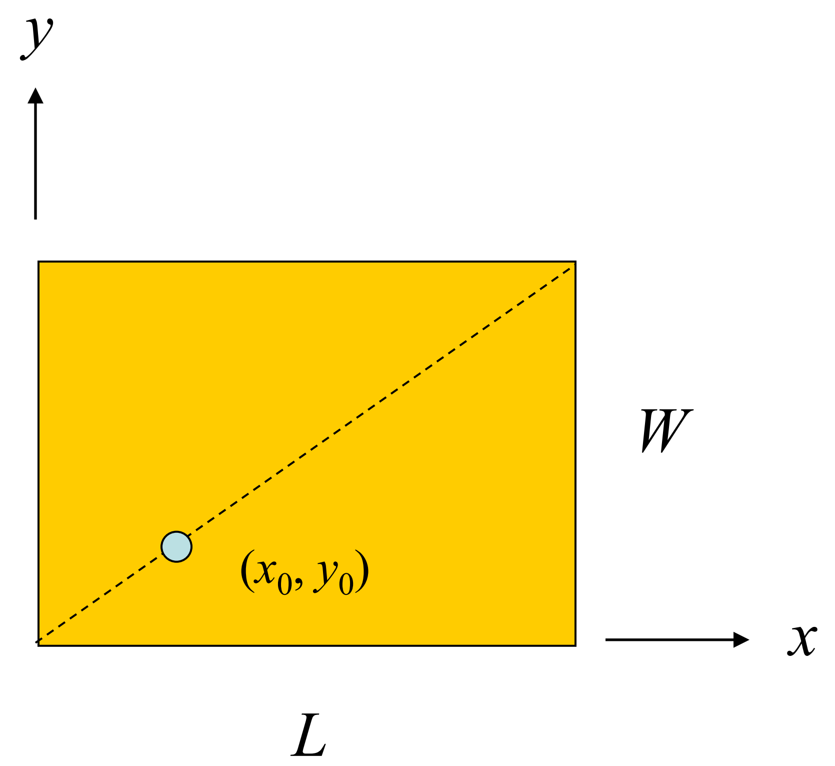
$$x_0 \approx y_0$$

Then $A_1^x \approx A_1^y \equiv A_1$

$$A_x = A_1 Z_{\text{in}}^x$$

$$A_y = A_1 Z_{\text{in}}^y$$

$$A_1 = \frac{1}{\cos\left(\frac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$



Amplitude of Patch Currents (cont.)

Because of the nearly equal dimensions and the feed along the diagonal, we also have:

$$R_{\text{in}}^{x} \approx R_{\text{in}}^{y} \equiv R$$

(The resistor in the circuit model for either mode is the same.)

R_{in}^{x} = resonant input resistance of the mode x, when excited by itself, and similarly for R_{in}^{y}

$\left(R_{\text{in}}^{x}\right.$ only depends on x_0, R_{in}^{y} only depends on $\left.y_0.\right)$

We then have:

$$A_x = A_2 \, \bar{Z}_{\text{in}}^{x}$$
$$A_y = A_2 \, \bar{Z}_{\text{in}}^{y}$$

where $A_2 \equiv A_1 R$

$$\Rightarrow A_2 = \frac{R}{\cos\left(\dfrac{\pi x_0}{L}\right)} \left[\frac{\omega \varepsilon_0 \varepsilon_r L}{j \pi h}\right]$$

Notation: The bar denotes impedances that are normalized by R.

Amplitude of Patch Currents (cont.)

The A_2 coefficient can be written as:

$$A_2 = \frac{R}{\cos\left(\frac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$

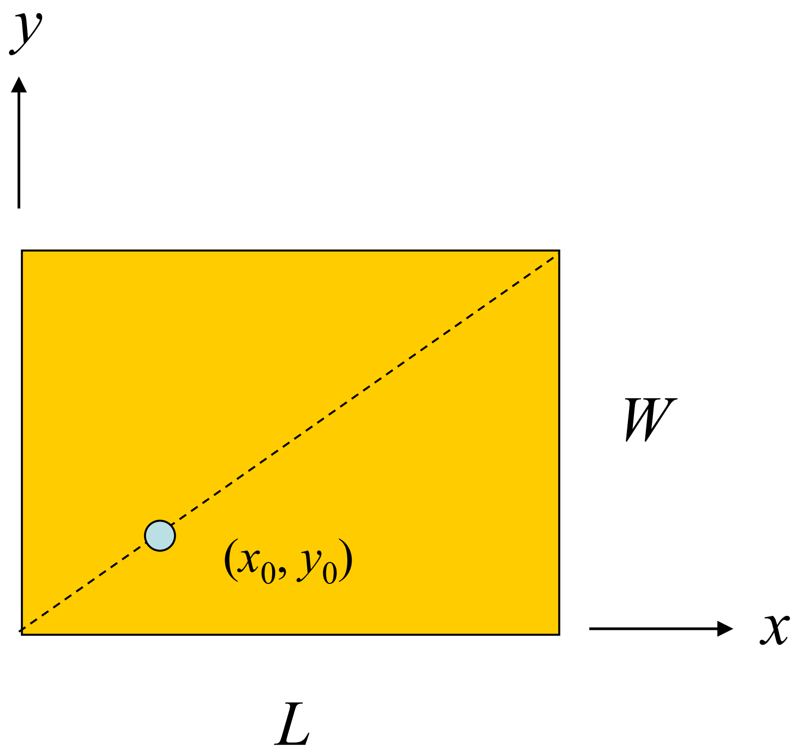
$$= \frac{R_{\text{edge}}\cos^2\left(\frac{\pi x_0}{L}\right)}{\cos\left(\frac{\pi x_0}{L}\right)}\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$

Note:
We can use the CAD formula for R_{edge}.

so

$$A_2 = R_{\text{edge}}\cos\left(\frac{\pi x_0}{L}\right)\left[\frac{\omega\varepsilon_0\varepsilon_r L}{j\pi h}\right]$$

Circular Polarization Condition



Let

$$L = W(1+\delta)$$

$$y_0 \approx x_0$$

The CP condition is

$$\frac{A_y}{A_x} = \pm j$$

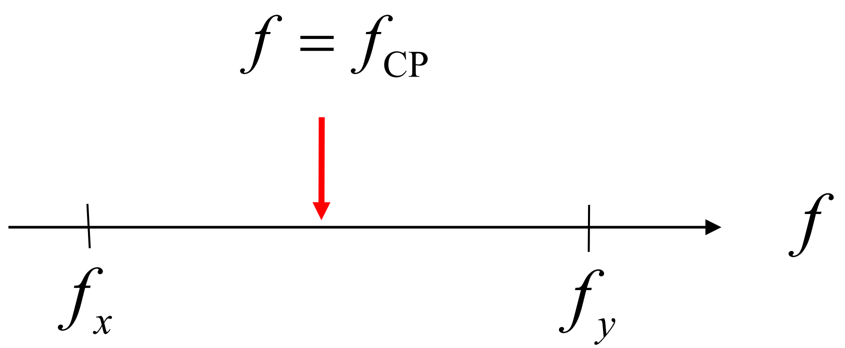
- for RHCP
+ for LHCP

A_x = amplitude of x mode current (TM_{10})

A_y = amplitude of y mode current (TM_{01})

Circular Polarization Condition (cont.)

The frequency f_{CP} is defined as the frequency for which we get CP at broadside.



We have, from the CAD circuit model:

$$\bar{Z}_{\mathrm{in}} = \frac{1}{1 + j2Q(f_r - 1)}$$

So, for each mode we have:

$$A_x = \frac{A_2}{1 + j2Q\left(f_{rx} - 1\right)}$$

$$A_y = \frac{A_2}{1 + j2Q\left(f_{ry} - 1\right)}$$

$$A_x = A_2\,\bar{Z}_{\mathrm{in}}^{x}$$

$$A_y = A_2\,\bar{Z}_{\mathrm{in}}^{y}$$

where

$$f_{rx} \equiv \frac{f}{f_x} \qquad f_{ry} \equiv \frac{f}{f_y}$$

f_x = resonance frequency of TM_{10} mode

f_y = resonance frequency of TM_{01} mode

Circular Polarization Condition (cont.)

LHCP:

$$\frac{A_y}{A_x} = +j$$

at $f = f_{\mathrm{CP}}$

Choose:

$$f_{rx} - 1 = \frac{1}{2Q}$$

$$f_{ry} - 1 = -\frac{1}{2Q}$$

Then we have:

$$A_x = \frac{A}{1+j}$$

$$A_y = \frac{A}{1-j}$$

$\Rightarrow$

$$\frac{A_y}{A_x} = \frac{1+j}{1-j} = \frac{\sqrt{2}e^{j\frac{\pi}{4}}}{\sqrt{2}e^{-j\frac{\pi}{4}}} = e^{j\frac{\pi}{2}} = j$$

(LHCP)

Circular Polarization Condition (cont.)

The frequency conditions for $f = f_{\text{CP}}$ can be written as:

$$f_{rx} - 1 = \frac{1}{2Q} \quad \Rightarrow \quad \frac{f_{\text{CP}}}{f_x} = 1 + \frac{1}{2Q} \quad \Rightarrow \quad \frac{f_x}{f_{\text{CP}}} \approx 1 - \frac{1}{2Q}$$

$$f_{ry} - 1 = -\frac{1}{2Q} \quad \Rightarrow \quad \frac{f_{\text{CP}}}{f_y} = 1 - \frac{1}{2Q} \quad \Rightarrow \quad \frac{f_y}{f_{\text{CP}}} \approx 1 + \frac{1}{2Q}$$

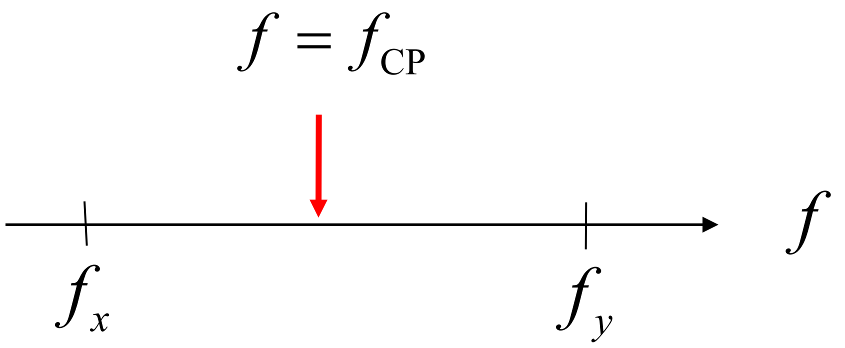
Note:

$$\frac{1}{1+x} \approx 1 - x$$

$$\frac{1}{1-x} \approx 1 + x$$

so

$$\frac{f_x + f_y}{f_{\text{CP}}} = 2$$



or

$$f_{\text{CP}} = \frac{1}{2}\left(f_x + f_y\right)$$

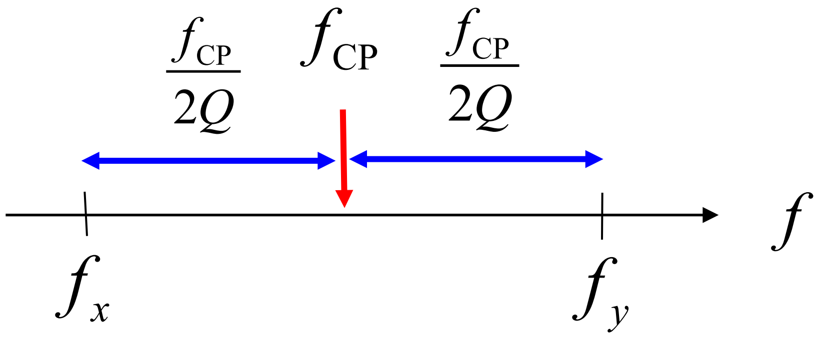
Circular Polarization Condition (cont.)

Also,
$$\frac{f_y}{f_{\mathrm{CP}}} - \frac{f_x}{f_{\mathrm{CP}}} = \left(1 + \frac{1}{2Q}\right) - \left(1 - \frac{1}{2Q}\right) = \frac{1}{Q}$$

Let $\Delta f \equiv f_y - f_x$

Then we have:
$$\frac{\Delta f}{f_{\mathrm{CP}}} = \frac{1}{Q}$$

Note:
For RHCP, the two frequencies are reversed.



Circular Polarization Condition (cont.)

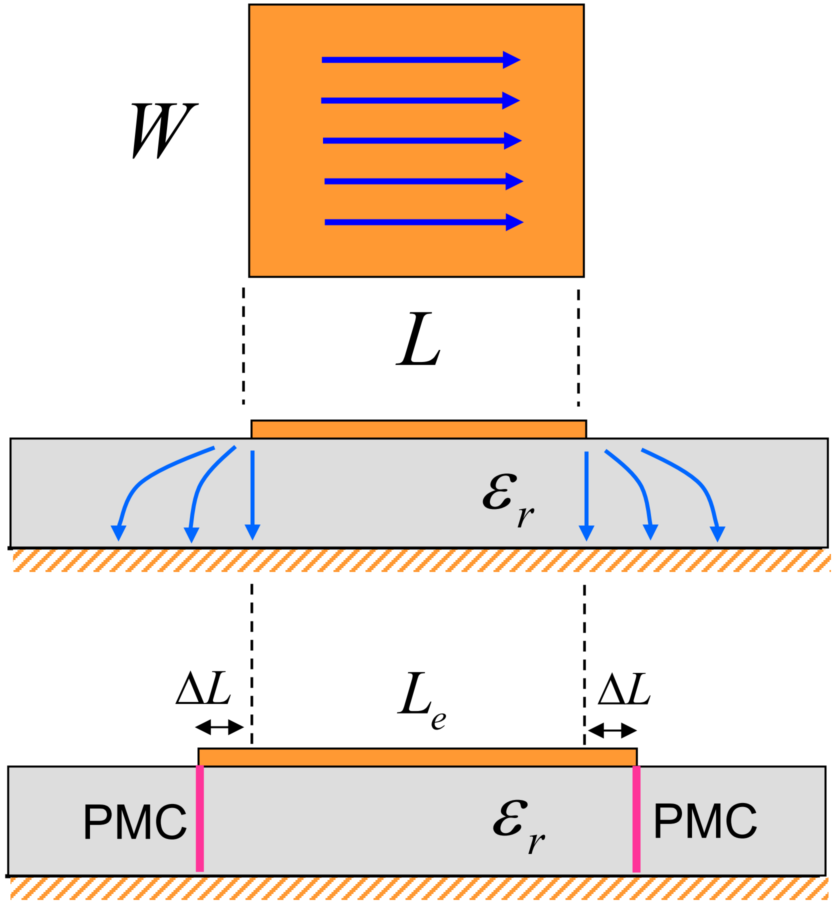
Summary of frequencies

$$f_x = f_{\mathrm{CP}}\left[1-\frac{1}{2Q}\right] \qquad f_y = f_{\mathrm{CP}}\left[1+\frac{1}{2Q}\right] \qquad \text{(LHCP)}$$

$$f_x = f_{\mathrm{CP}}\left[1+\frac{1}{2Q}\right] \qquad f_y = f_{\mathrm{CP}}\left[1-\frac{1}{2Q}\right] \qquad \text{(RHCP)}$$

f_{CP} = frequency for which we get CP at broadside.

Patch Dimensions for CP



$$\Delta L = 0.412h \left[\frac{\left(\frac{W}{h}\right) + 0.262}{\left(\frac{W}{h}\right) + 0.813} \right] \left[\frac{\varepsilon_r^{\text{eff}} + 0.300}{\varepsilon_r^{\text{eff}} - 0.258} \right]$$

$$\varepsilon_r^{\text{eff}} = \left(\frac{\varepsilon_r + 1}{2}\right) + \left(\frac{\varepsilon_r - 1}{2}\right) \frac{1}{\sqrt{1 + 12\left(\frac{h}{W}\right)}}$$

$$L_e = L + 2\Delta L$$

$$\Delta L = f\left(h, \varepsilon_r, W\right)$$ (Hammerstad formula)

Patch Dimensions for CP (cont.)

$$f_x = f_{\mathrm{CP}}\left[1 \mp \frac{1}{2Q}\right] \qquad f_y = f_{\mathrm{CP}}\left[1 \pm \frac{1}{2Q}\right]$$

Top sign: LHCP
Bottom sign: RHCP

Define:

$$k_{0x} \equiv 2\pi\, f_x \sqrt{\mu_0 \varepsilon_0}$$
$$k_{0y} \equiv 2\pi\, f_y \sqrt{\mu_0 \varepsilon_0}$$

(These are known wavenumbers, since the frequencies are known.)

$$k_{0x} L_e \sqrt{\varepsilon_r} = \pi$$
$$k_{0y} W_e \sqrt{\varepsilon_r} = \pi$$

(resonance condition)

$$L_e = \frac{\pi}{k_{0x}\sqrt{\varepsilon_r}}$$

$$W_e = \frac{\pi}{k_{0y}\sqrt{\varepsilon_r}}$$

Now we know the effective dimensions of the patch.

Patch Dimensions for CP (cont.)

For the physical patch dimensions we have:

$$L = L_e - 2\Delta L\left(h, \varepsilon_r, W\right)$$

$$W = W_e - 2\Delta W\left(h, \varepsilon_r, L\right)$$

Note: For ΔW, we use the same formula as ΔL , but replace $W \to L$.

$$\Delta L = 0.412h \left[\frac{\left(\frac{W}{h}\right) + 0.262}{\left(\frac{W}{h}\right) + 0.813} \right] \left[\frac{\varepsilon_r^{\text{eff}} + 0.300}{\varepsilon_r^{\text{eff}} - 0.258} \right]$$

$$\Delta W = 0.412h \left[\frac{\left(\frac{L}{h}\right) + 0.262}{\left(\frac{L}{h}\right) + 0.813} \right] \left[\frac{\varepsilon_r^{\text{eff}} + 0.300}{\varepsilon_r^{\text{eff}} - 0.258} \right]$$

Physical Dimensions for CP (cont.)

Summary of Design Equations

Patch Dimensions

$$L = L_e - 2\Delta L$$
$$W = W_e - 2\Delta W$$

where

$$L_e = \frac{\pi}{k_{0x}\sqrt{\varepsilon_r}} \qquad k_{0x} \equiv 2\pi f_x \sqrt{\mu_0 \varepsilon_0}$$

$$W_e = \frac{\pi}{k_{0y}\sqrt{\varepsilon_r}} \qquad k_{0y} \equiv 2\pi f_y \sqrt{\mu_0 \varepsilon_0}$$

$$f_x = f_{\text{CP}}\left[1 \mp \frac{1}{2Q}\right] \qquad f_y = f_{\text{CP}}\left[1 \pm \frac{1}{2Q}\right]$$

Top sign: LHCP, Bottom sign: RHCP

Note: The fringing length ΔL depends on W, and the fringing length ΔW depends on L.

Physical Dimensions for CP (cont.)

Iterative Design Process:

- Neglect fringing extensions, and take $L = L_e$ and $W = W_e$.
- Solve for the fringing extensions ΔL and ΔW using the Hammerstad formula.
- Solve for $L = L_e - 2\Delta L$ and $W = W_e - 2\Delta W$.
- Repeat from step 2 if necessary.

Input Impedance of CP Patch

$$Z_{\text{in}}^{\text{CP}}(f) = Z_{\text{in}}^{x}(f) + Z_{\text{in}}^{y}(f) = \frac{R}{1 + j2Q\left(f_{rx} - 1\right)} + \frac{R}{1 + j2Q\left(f_{ry} - 1\right)}$$

$$\left(R_{\text{in}}^{x} \approx R_{\text{in}}^{y} \equiv R\right)$$

At f_{CP}: $f_{rx} - 1 = 1/(2Q)$ and $f_{ry} - 1 = -1/(2Q)$ (LHCP)

so

$$Z_{\text{in}}^{\text{CP}}\left(f_{\text{CP}}\right) = \frac{R}{1+j} + \frac{R}{1-j} = \frac{R(1-j) + R(1+j)}{(1+j)(1-j)} = \frac{R(1-j) + R(1+j)}{2} = R$$

or $Z_{\text{in}} = R$

Recall: $R = R^x = R$ for TM_{10} mode

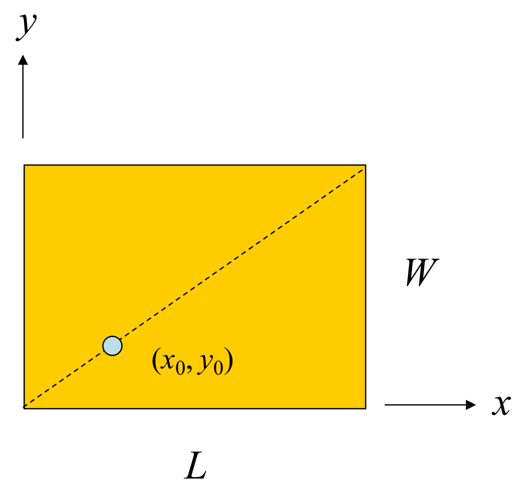
The CP frequency f_{CP} is also the resonance frequency where the input impedance is real (if we neglect the probe inductance).

Input Impedance of CP Patch (cont.)

Hence, at the resonance (CP) frequency f_{CP} we have

$$Z_{\mathrm{in}}^{\mathrm{CP}} = R_{\mathrm{edge}} \cos^2\left(\frac{\pi x_0}{L}\right)$$

$R_{\mathrm{edge}} = R_{\mathrm{edge}}$ for TM_{10} mode

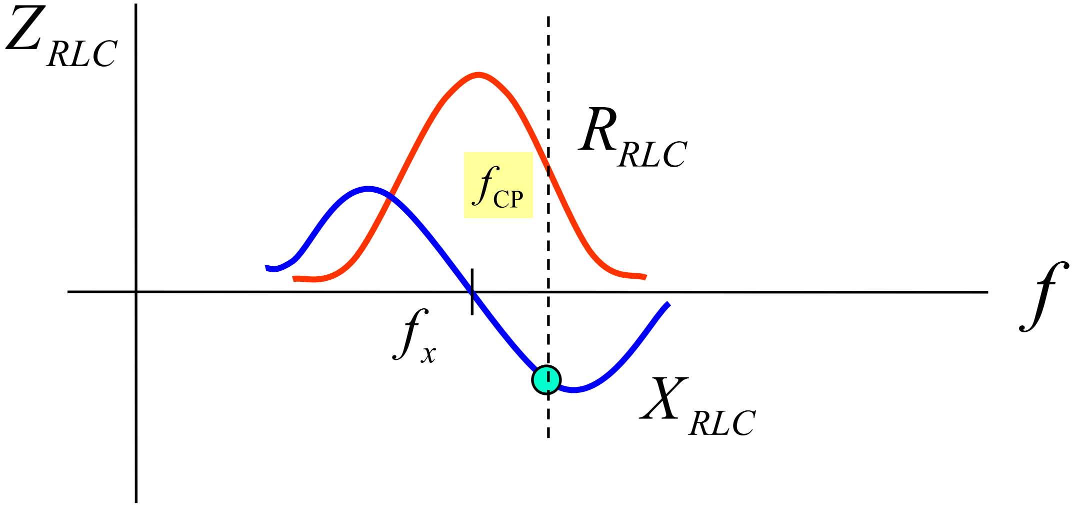


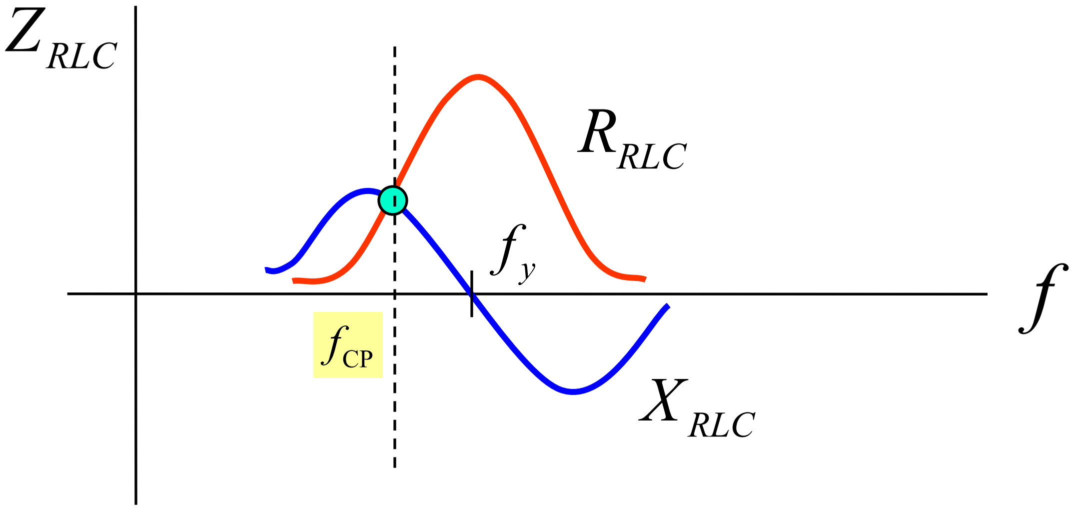
Note: We have a CAD formula for R_{edge}.

The feed position x_0 can be chosen to give the desired input resistance at the resonance frequency f_{CP}.

Input Impedance of CP Patch (cont.)

A physical picture of the frequencies (LHCP):





$$Z_{\text{in}}^{x}\left(f_{\text{CP}}\right)=\frac{R}{1+j}$$

$$X_{\text{in}}^{x}\left(f_{\text{CP}}\right)=-R_{\text{in}}^{x}\left(f_{\text{CP}}\right)$$

$$Z_{\text{in}}^{y}\left(f_{\text{CP}}\right)=\frac{R}{1-j}$$

$$X_{\text{in}}^{y}\left(f_{\text{CP}}\right)=R_{\text{in}}^{y}\left(f_{\text{CP}}\right)$$

CP (Axial Ratio) Bandwidth

We now examine the frequency dependence of the term A_y / A_x.

$$A_x = \frac{A_2}{1 + j2Q\left(f_{rx} - 1\right)}$$

$$A_y = \frac{A_2}{1 + j2Q\left(f_{ry} - 1\right)}$$

$$\Rightarrow \quad \frac{A_y}{A_x} = \frac{1 + j2Q(f_{rx} - 1)}{1 + j2Q(f_{ry} - 1)}$$

where

$$f_{rx} \equiv \frac{f}{f_x} = \frac{f}{f_{\mathrm{CP}}\left(1 - \frac{1}{2Q}\right)} = f_r \left(\frac{1}{1 - \frac{1}{2Q}} \right) \approx f_r \left(1 + \frac{1}{2Q}\right)$$

$$f_r \equiv \frac{f}{f_{\mathrm{CP}}}$$

This is the ratio of the operating frequency to the CP frequency.

(LHCP)

CP Bandwidth (cont.)

Then $$f_{rx} \approx f_r\left(1+\frac{1}{2Q}\right)$$

$$f_{rx} = \frac{f}{f_x} = \frac{f}{f_{\text{CP}}\left(1-\frac{1}{2Q}\right)} = f_r\left(\frac{1}{1-\frac{1}{2Q}}\right) \approx f_r\left(1+\frac{1}{2Q}\right)$$

Similarly, $$f_{ry} \approx f_r\left(1-\frac{1}{2Q}\right)$$

$$f_{ry} = \frac{f}{f_y} = \frac{f}{f_{\text{CP}}\left(1+\frac{1}{2Q}\right)} = f_r\left(\frac{1}{1+\frac{1}{2Q}}\right) \approx f_r\left(1-\frac{1}{2Q}\right)$$

CP Bandwidth (cont.)

Hence, we have:

$$\frac{A_y}{A_x} = \frac{1 + j2Q\left(f_{rx} - 1\right)}{1 + j2Q\left(f_{ry} - 1\right)} = \frac{1 + j2Q\left(f_r\left(1 + \frac{1}{2Q}\right) - 1\right)}{1 + j2Q\left(f_r\left(1 - \frac{1}{2Q}\right) - 1\right)}$$

Note: $f_r = 1 \quad \Rightarrow \quad \frac{A_y}{A_x} = \frac{1 + j}{1 - j} = j \quad (\text{LHCP})$

Let $x \equiv 2Q\left(f_r - 1\right)$ $\qquad f_r \equiv \frac{f}{f_{\text{CP}}}$

Then

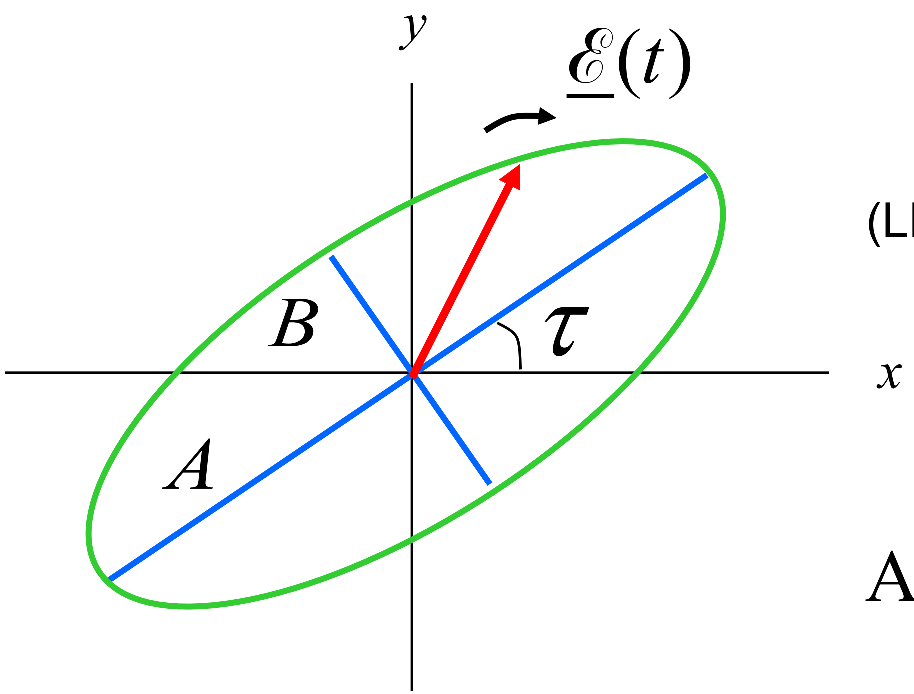
$$\frac{A_y}{A_x} = \frac{1 + j(f_r + x)}{1 - j(f_r - x)} \approx \frac{1 + j(1 + x)}{1 - j(1 - x)} \qquad (f_r \approx 1)$$

CP Bandwidth (cont.)

$$\frac{A_y}{A_x} = \frac{1 + j(1+x)}{1 - j(1-x)}$$

$$x \equiv 2Q\left(f_r - 1\right)$$

$$f_r \equiv \frac{f}{f_{\text{CP}}}$$



(LHCP)

$$\text{AR} \equiv \frac{A}{B}$$

CP Bandwidth (cont.)

From ECE 6340: $\mathrm{AR} = \cot\xi$

where

$$\sin 2\xi = \sin(2\gamma)\sin\phi \qquad \text{with} \qquad -\frac{\pi}{2} \le 2\xi \le \frac{\pi}{2}$$

$$\gamma = \tan^{-1}\left|\frac{A_y}{A_x}\right|$$

$$\phi = \arg\left(\frac{A_y}{A_x}\right)$$

In our case:

$$\gamma = \tan^{-1}\sqrt{\frac{1+(1+x)^2}{1+(1-x)^2}}$$

$$\phi = \tan^{-1}(1+x) - \tan^{-1}\left(-(1-x)\right) = \tan^{-1}(1+x) + \tan^{-1}(1-x)$$

CP Bandwidth (cont.)

Set $\mathrm{AR} = \sqrt{2} \quad \left(\mathrm{AR}_{\mathrm{dB}} = 20\log_{10}\left(\mathrm{AR}\right) = 3\ \mathrm{dB} \right)$

From a numerical solution: $x = \pm 0.348$

CP Bandwidth (cont.)

Hence, we have: $2Q\left(f_r - 1\right) = \pm 0.348$

$$f_r = 1 \pm \frac{0.348}{2Q}$$

so

$$f_r^+ = 1 + \frac{0.348}{2Q}$$

$$f_r^- = 1 - \frac{0.348}{2Q}$$

Therefore, $\Delta f_r = f_r^+ - f_r^- = \dfrac{0.348}{Q}$

$$\mathrm{BW}^{\mathrm{AR\ CP}} \equiv \frac{f^+ - f^-}{f_{\mathrm{CP}}} = f_r^+ - f_r^-$$

Hence, we have:

$$\mathrm{BW}^{\mathrm{AR\ CP}} = \frac{0.348}{Q}$$

Impedance Bandwidth

$$Z_{\text{in}}(f) = \frac{R}{1 + j2Q(f_{rx} - 1)} + \frac{R}{1 + j2Q(f_{ry} - 1)}$$

$$= \frac{R}{1 + j2Q\left(f_r\left(1 + \frac{1}{2Q}\right) - 1\right)} + \frac{R}{1 + j2Q\left(f_r\left(1 - \frac{1}{2Q}\right) - 1\right)}$$

(LHCP)

$$= \frac{R}{1 + j(f_r + x)} + \frac{R}{1 - j(f_r - x)}$$

$$\approx \frac{R}{1 + j(1 + x)} + \frac{R}{1 - j(1 - x)} \qquad (\text{for } f_r \approx 1)$$

where $x \equiv 2Q(f_r - 1)$

Note: At $x = 0$ we have $Z_{\text{in}} = \frac{R}{1+j} + \frac{R}{1-j} = R$

Impedance Bandwidth (cont.)

$$\bar{Z}_{\text{in}} \equiv \frac{Z_{\text{in}}}{R} = \frac{1}{1+j(1+x)} + \frac{1}{1-j(1-x)}$$

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_{\text{in}} - R}{Z_{\text{in}} + R} = \frac{\bar{Z}_{\text{in}} - 1}{\bar{Z}_{\text{in}} + 1}$$

$$S = \text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Set $S = S_0 = 2$ (bandwidth limits) $\Rightarrow$ $|\Gamma| = \frac{1}{3}$ (-9.5 dB)

$$\Rightarrow \left| \frac{\frac{1}{1+j(1+x)} + \frac{1}{1-j(1-x)} - 1}{\frac{1}{1+j(1+x)} + \frac{1}{1-j(1-x)} + 1} \right| = \frac{1}{3} \Rightarrow x_0 = \pm\sqrt{2}$$

(derivation omitted)

Impedance Bandwidth (cont.)

Hence $$x \equiv 2Q\left(f_r - 1\right) = x_0 = \pm\sqrt{2}$$

so

$$f_r = 1 \pm \frac{\sqrt{2}}{2Q}$$

The band edges (in normalized frequency) are then

$$f_r^+ = 1 + \frac{1}{\sqrt{2}Q}$$

$$f_r^- = 1 - \frac{1}{\sqrt{2}Q}$$

Impedance Bandwidth (cont.)

$$\mathrm{BW}^{\mathrm{Imp\ CP}} \equiv \frac{f^{+} - f^{-}}{f_{\mathrm{CP}}} = f_r^{+} - f_r^{-}$$

Hence

$$\mathrm{BW}^{\mathrm{Imp\ CP}} = 2\left(\frac{1}{\sqrt{2}Q}\right)$$

Hence

$$\mathrm{BW}^{\mathrm{Imp\ CP}} = \frac{\sqrt{2}}{Q}$$

Summary

CP antenna

$$\mathrm{BW}^{\mathrm{AR\ CP}} = \frac{0.348}{Q}$$

$$\mathrm{BW}^{\mathrm{Imp\ CP}} = \frac{\sqrt{2}}{Q} = \frac{1.414}{Q}$$

Linear antenna

$$\mathrm{BW}^{\mathrm{Imp\ Lin}} = \frac{1}{\sqrt{2}Q} = \frac{0.707}{Q}$$