ECE 6345

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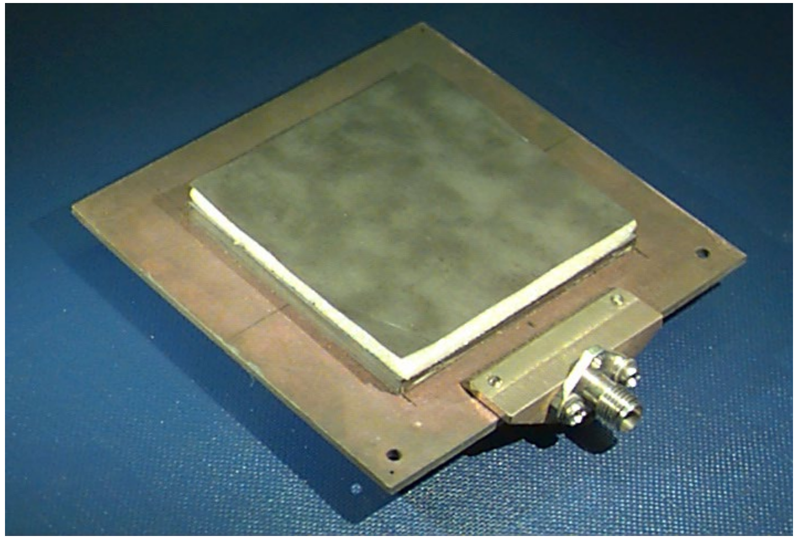


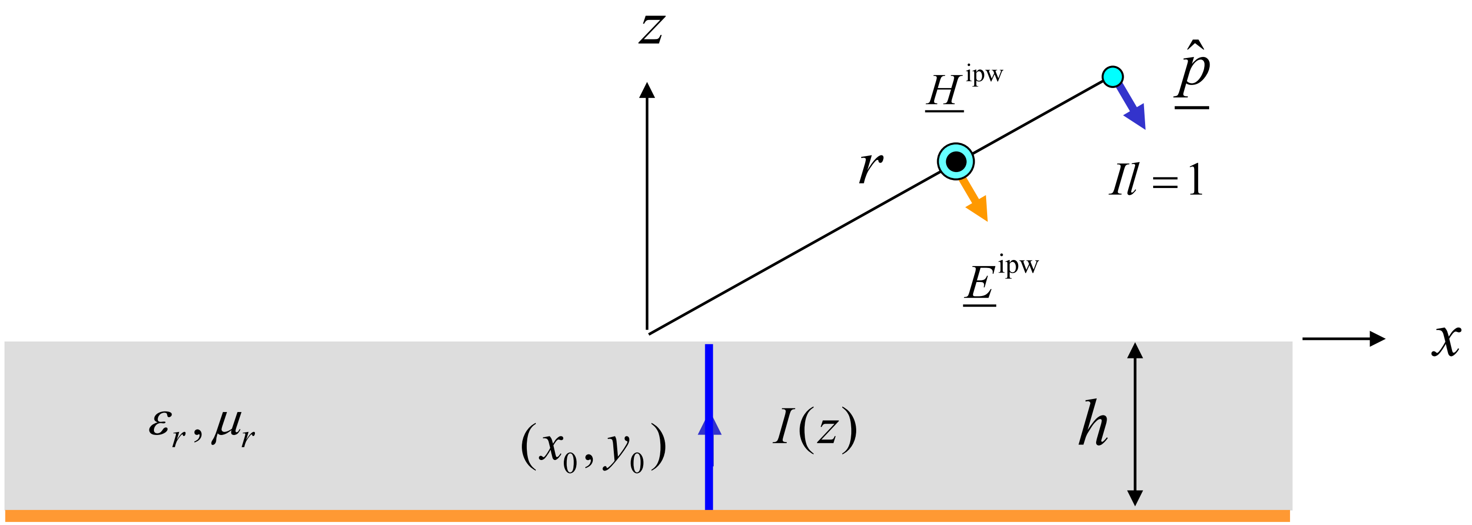
Notes 9

Overview

In this set of notes we calculate the far field of a vertical probe feed.

- We first formulate the far-field radiation from a uniform probe current of fixed (arbitrary) amplitude.
- The probe current is then related to the patch input impedance.

Radiation from Probe Current



The origin is at the center of the patch (which has been removed).

$$E_\phi^{\mathrm{FF}} = 0$$

This follows from reciprocity, since

$$\underline{\hat{\phi}} \cdot \underline{\hat{z}} = 0$$

$$E_\theta^{\mathrm{FF}}(r,\theta,\phi) = \langle b, a \rangle = \int_{-h}^{0} I(z)\, E_z^{\mathrm{pw}}(z)\, dz$$

$$E_z^{\mathrm{pw}}(x,y,z) = E_z^{\mathrm{ipw}}(x,y,z) + E_z^{\mathrm{rpw}}(x,y,z)$$

Radiation from Probe Current (cont.)

Inside the substrate we have:

$$E_z = \frac{1}{j\omega\varepsilon_1}\left(\nabla \times \underline{H}\right)\cdot \underline{\hat{z}} \qquad \varepsilon_1 = \varepsilon_0\varepsilon_r$$

$$= \frac{1}{j\omega\varepsilon_1}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

Transverse variation:

$$\psi_t\left(x, y\right) = e^{j\left(k_x x + k_y y\right)}$$

Hence, we have:

$$E_z^b = \frac{1}{j\omega\varepsilon_1}\left[jk_x H_y^{\text{pw}} - jk_y H_x^{\text{pw}} \right]$$

Radiation from Probe Current (cont.)

For the magnetic field inside the substrate:

$$\underline{H}^{\text{ipw}} = \underline{\hat{\phi}}\left(-\frac{E_\theta^{\text{ipw}}}{\eta_0}\right) = \underline{\hat{\phi}}\left(-\frac{E_0}{\eta_0}\right)e^{j\left(k_x x + k_y y + k_z z\right)}$$

From this we have:

$$H_x^{\text{ipw}} = -\frac{E_0}{\eta_0}e^{j\left(k_x x + k_y y + k_z z\right)}\left(-\sin\phi\right) \qquad \left(\underline{\hat{\phi}}\cdot\underline{\hat{x}} = -\sin\phi\right)$$

$$H_y^{\text{ipw}} = -\frac{E_0}{\eta_0}e^{j\left(k_x x + k_y y + k_z z\right)}\cos\phi \qquad \left(\underline{\hat{\phi}}\cdot\underline{\hat{y}} = \cos\phi\right)$$

Radiation from Probe Current (cont.)

For the total (incident plus reflected) plane-wave field we have (for the x component):

$$H_x^{\text{pw}}(x_0, y_0, 0) = H_x^{\text{ipw}}(x_0, y_0, 0)\left(1-\Gamma^{\text{TM}}\right) = \left[\frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \sin\phi\right]\left(1-\Gamma^{\text{TM}}\right)$$

Denote:

$$H_x^{\text{pw}}(x_0, y_0, z) = f(z) = A\cos\left(k_{z1}(z+h)\right)$$

where

$$k_{z1} = \left(k_1^2 - k_x^2 - k_y^2\right)^{1/2} = \left(k_1^2 - k_0^2 \sin^2\theta\right)^{1/2} = k_0\sqrt{\mu_r \varepsilon_r - \sin^2\theta} = k_0 N_1(\theta)$$

Setting $z = 0$ we have:

$$A = \frac{H_x^{\text{pw}}(x_0, y_0, 0)}{\cos\left(k_{z1}h\right)}$$

Hence, we have:

$$H_x^{\text{pw}}(x_0, y_0, z) = H_x^{\text{pw}}(x_0, y_0, 0)\left[\frac{\cos\left(k_{z1}(z+h)\right)}{\cos\left(k_{z1}h\right)}\right]$$

Radiation from Probe Current (cont.)

We then have:

$$H_x^{\text{pw}}\left(x_0,y_0,z\right)=\frac{E_0}{\eta_0}e^{j\left(k_x x_0+k_y y_0\right)}\sin\phi\left(1-\Gamma^{\text{TM}}\right)\sec\left(k_{z1}h\right)\cos k_{z1}(z+h)$$

Similarly,

$$H_y^{\text{pw}}\left(x_0,y_0,z\right)=\frac{E_0}{\eta_0}e^{j\left(k_x x_0+k_y y_0\right)}\left(-\cos\phi\right)\left(1-\Gamma^{\text{TM}}\right)\sec\left(k_{z1}h\right)\cos k_{z1}(z+h)$$

Next, use

$$E_z^{\text{pw}}=\frac{1}{\omega\varepsilon_1}\left[k_x H_y^{\text{pw}}-k_y H_x^{\text{pw}}\right]$$

$$k_x=k_0\sin\theta\cos\phi$$
$$k_y=k_0\sin\theta\sin\phi$$

Radiation from Probe Current (cont.)

We then have:

$$E_z^{\text{pw}} = \frac{1}{\omega\varepsilon_1}\frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{\text{TM}}\right) \sec\left(k_{z1} h\right) \cos k_{z1}(z+h) \left[k_0 \sin\theta \left(-\cos^2\phi - \sin^2\phi \right) \right]$$

or

$$\begin{aligned} E_z^{\text{pw}} &= -\frac{1}{\omega\varepsilon_1}\frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{\text{TM}}\right) \sec\left(k_{z1} h\right) \cos k_{z1}(z+h)\; k_0 \sin\theta \\ &= -\frac{1}{\varepsilon_r} E_0\, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{\text{TM}}\right) \sec\left(k_{z1} h\right) \cos k_{z1}(z+h)\; \sin\theta \end{aligned}$$

$$\left(\frac{k_0}{\omega\varepsilon_0} = \eta_0 \right)$$

Radiation from Probe Current (cont.)

The far field pattern is then:

$$E_\theta^{\mathrm{FF}}(r,\theta) = \frac{-1}{\varepsilon_r} E_0 \, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{\mathrm{TM}}\right) \sec\left(k_{z1} h\right) \sin\theta \int_{-h}^{0} I(z) \cos\left(k_{z1}(z+h)\right) dz$$

Assume $I(z) = I_0$

$$\begin{aligned}
\int_{-h}^{0} I(z)\cos\left(k_{z1}(z+h)\right) dz &= I_0 \int_{-h}^{0} \cos\left(k_{z1}(z+h)\right) dz \\
&= I_0 \int_{0}^{h} \cos\left(k_{z1} u\right) du \qquad \left(u = z+h\right) \\
&= \frac{I_0}{k_{z1}} \sin\left(k_{z1} h\right) \\
&= I_0 h \, \mathrm{sinc}\left(k_{z1} h\right)
\end{aligned}$$

Radiation from Probe Current (cont.)

Hence, we have:

$$E_{\theta}^{\mathrm{FF}}(r,\theta)=\frac{-1}{\varepsilon_r}E_0\,e^{j\left(k_x x_0+k_y y_0\right)}\left(1-\Gamma^{\mathrm{TM}}\right)\sec\left(k_{z1}h\right)\left[\left(I_0 h\right)\mathrm{sinc}\left(k_{z1}h\right)\,\sin\theta\right]$$

or

$$E_{\theta}^{\mathrm{FF}}(r,\theta)=\left(I_0 h\right)\left(\frac{-1}{\varepsilon_r}\right)E_0\,e^{j\left(k_x x_0+k_y y_0\right)}\left(1-\Gamma^{\mathrm{TM}}\right)\mathrm{tanc}\left(k_0 h N_1(\theta)\right)\sin\theta$$

where

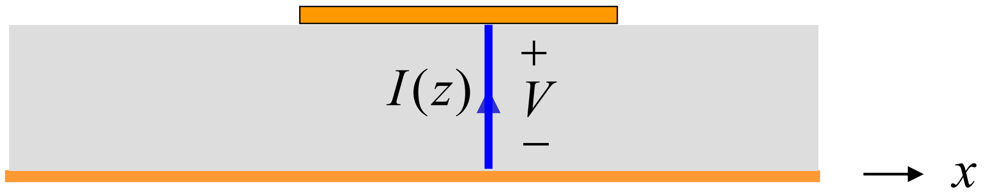
$$\mathrm{tanc}\left(x\right)\equiv\frac{\tan\left(x\right)}{x}\qquad\qquad\left(\sec\left(x\right)\mathrm{sinc}\left(x\right)=\mathrm{tanc}\left(x\right)\right)$$

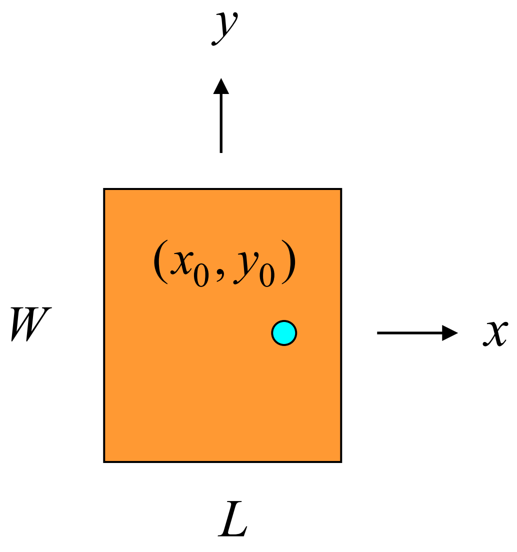
Calculation of Probe Current

We want to find the relation between these two.

$$J_{sx}(x,y) = A_{10}^{J}\cos\left(\frac{\pi x}{L}\right)$$

$$I(z) = I_0$$





$$V = Z_{\text{in}} I_0 = -h\,E_z(x_0, y_0)$$

Z_{in} = input impedance of dominant mode (known).

Hence, we have:

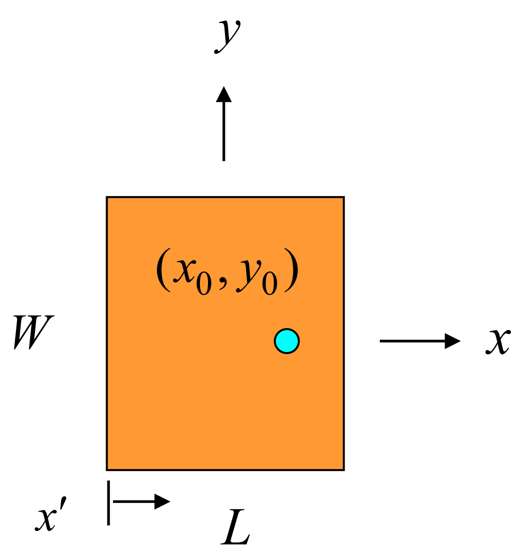
$$E_z(x_0, y_0) = -\frac{Z_{\text{in}} I_0}{h}$$

Calculation of Probe Current (cont.)

Assume: $$E_z^{10}(x,y) = -A_{10}^E \sin\left(\frac{\pi x}{L}\right)$$ $$\left(E_z^{10}(x',y') = A_{10}^E \cos\left(\frac{\pi x'}{L}\right) \right)$$

Then

$$A_{10}^E = -\frac{E_z^{10}(x_0,y_0)}{\sin\left(\frac{\pi x_0}{L}\right)} = \frac{Z_{\text{in}}\, I_0}{h \sin\left(\frac{\pi x_0}{L}\right)}$$



Therefore, we have:

$$E_z^{10}(x,y) = \left[\frac{-Z_{\text{in}} I_0}{h \sin\left(\frac{\pi x_0}{L}\right)} \right] \sin\left(\frac{\pi x}{L}\right)$$

Calculation of Probe Current (cont.)

From previous calculations,

$$\underline{J}_s = -\underline{\hat{z}} \times \underline{H} = \frac{1}{j\omega\mu} \nabla E_z$$

$$\underline{H} = -\frac{1}{j\omega\mu} \nabla \times \underline{E}$$
$$= -\frac{1}{j\omega\mu} \nabla \times \left(\underline{\hat{z}} E_z(x,y) \right)$$
$$= -\frac{1}{j\omega\mu} \left(-\underline{\hat{z}} \times \nabla E_z(x,y) \right)$$

Hence, we have:

$$J_{sx}^{10} = \frac{1}{j\omega\mu} \left(\frac{\pi}{L} \right) \left[\frac{-Z_{\text{in}} I_0}{h \sin\left(\frac{\pi x_0}{L} \right)} \right] \cos\left(\frac{\pi x}{L} \right) = A_{10}^J \cos\left(\frac{\pi x}{L} \right)$$

so that

$$A_{10}^J = -\frac{1}{j\omega\mu} \left(\frac{\pi}{L} \right) \left[\frac{Z_{\text{in}} I_0}{h \sin\left(\frac{\pi x_0}{L} \right)} \right]$$

or

$$\frac{A_{10}^J}{I_0} = -\frac{1}{j\omega\mu} \left(\frac{\pi}{L} \right) \left[\frac{Z_{\text{in}}}{h \sin\left(\frac{\pi x_0}{L} \right)} \right]$$

Calculation of Probe Current (cont.)

At resonance,

$$Z_{\text{in}} = R_{\text{edge}} \sin^2\left(\frac{\pi x_0}{L}\right)$$

(The origin is at the center of the patch here.)

Hence, we have:

$$\frac{A_{10}^J}{I_0} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{R_{\text{edge}}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right)$$

Calculation of Probe Current (cont.)

We can also write this as

$$\frac{I_{\text{patch}}}{I_0} = \frac{W A_{10}^{J}}{I_0} = -\frac{\pi}{j\omega\mu}\left(\frac{W}{L}\right)\left[\frac{R_{\text{edge}}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right) = j\left(\frac{\pi}{\mu_r k_0 \eta_0}\right)\left(\frac{W}{L}\right)\left[\frac{R_{\text{edge}}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right)$$

I_{patch} is the total current (amps) flowing across the center of the patch.

We then have

$$\frac{I_{\text{patch}}}{I_0} = j\left(\frac{\pi}{\mu_r \eta_0}\right)\left(\frac{1}{k_0 h}\right)\left(\frac{W}{L}\right) R_{\text{edge}} \sin\left(\frac{\pi x_0}{L}\right)$$

Calculation of Probe Current (cont.)

Final result:

$$\frac{I_{\text{patch}}}{I_0} = j\pi \frac{R_{\text{edge}}}{\mu_r \eta_0} \left(\frac{1}{k_0 h} \right) \left(\frac{W}{L} \right) \sin\left(\frac{\pi x_0}{L} \right)$$

Note: For a thin substrate, the patch current is much larger than the probe current.

For a fixed patch current:

$$I_0 \propto k_0 h \quad \Rightarrow \quad I_0 h \propto \left(k_0 h\right)^2$$

(The term $I_0 h$ is a measure of how strong the probe radiates.)

The probe radiation gets large quickly as the substrate gets thicker!

Summary

$$E_\theta^{\mathrm{FF}}(r,\theta) = \left(I_0 h\right)\left(\frac{-1}{\varepsilon_r}\right) E_0\, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{\mathrm{TM}}\right) \operatorname{tanc}\left(k_0 h N_1(\theta)\right) \sin\theta$$

$$\frac{A_{10}^J}{I_0} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{\mathrm{in}}}{h\sin\left(\dfrac{\pi x_0}{L}\right)}\right]$$

Assumption: $J_{sx}(x,y) = A_{10}^J \cos\left(\dfrac{\pi x}{L}\right)$