ECE 6345
Spring 2024

Homework 5

1) A rectangular microstrip antenna is printed on a lossless substrate having a substrate relative permittivity $\varepsilon_r = 2.2$. The aspect ratio of the patch is $W/L = 1.5$. The antenna is operated at the resonant frequency (assume that fringing may be ignored, so the length of the patch is one-half of a wavelength in the dielectric). Plot the exact Q_{sp} versus the normalized substrate thickness h/λ_0 over the range $0 < h/\lambda_0 < 0.1$. On the same graph, add a plot of the CAD result for Q_{sp} (which involves the CAD formula for p). The exact Q_{sp} is based on the exact P_{sp}, which comes from the exact p factor (which must be found from a double integration in θ and ϕ), and also the exact $P_{\text{sp}}^{\text{dip}}$ (which comes from a single integration in θ).

   Repeat for a substrate relative permittivity of 10.8.

2) Consider the rectangular patch of Prob. 1. Plot the exact directivity (dB) versus the normalized substrate thickness h/λ_0 over the range $0 < h/\lambda_0 < 0.1$. On the same graph, add a plot of the CAD formula for the directivity (which uses the CAD formula for p). The exact directivity will involve the exact p (which involves a double integration in θ and ϕ) as well as the exact directivity of the dipole, which involves the exact power density radiated at broadside by the dipole and the exact space-wave power radiated by the dipole (which comes from a single integration in θ).

   Repeat for a substrate relative permittivity of 10.8.

3) A circular microstrip antenna is printed on a lossless substrate having a substrate relative permittivity $\varepsilon_r = 2.2$. The antenna is operated at the resonance frequency of the TM_{011} mode. Ignore fringing, so that

   $k_1 a = x'_{11} = 1.84118$ (and therefore $a = 0.29303\,\lambda_0 / \sqrt{\varepsilon_r}$).

   Plot the exact Q_{sp} versus the normalized substrate thickness h/λ_0 over the range $0 < h/\lambda_0 < 0.1$. On the same graph, add a plot of the CAD formula for Q_{sp} (which uses the CAD formula for p_c). The exact Q_{sp} for the circular patch will use the exact p_c, which will involve a single numerical integration in θ.

Repeat for a substrate relative permittivity of 10.8.

4) Consider the circular patch of Prob. 3. Plot the exact directivity (dB) versus the normalized substrate thickness h / λ_0 over the range $0 < h / \lambda_0 < 0.1$. On the same graph, add a plot of the CAD formula for the directivity (which uses the CAD formula for p_c). The exact directivity for the circular patch will use the exact p_c, which will involve a single numerical integration in θ.

   Repeat for a substrate relative permittivity of 10.8.

5) Consider the same patch as in Prob. 1. Plot the exact surface-wave radiation efficiency of the patch, together with the CAD formula for the surface-wave radiation efficiency of the patch. (In the CAD formula, the surface-wave efficiency of the patch is taken to be the same as that of the infinitesimal dipole.) For the exact radiation efficiency of the patch, use the two paths shown on slide 12 of Notes 24 to calculate the space-wave power and the total radiated (space-wave + surface-wave) power. Plot versus the normalized thickness of the substrate h / λ_0 over the range $0 < h / \lambda_0 < 0.1$. (Note: The surface-wave radiation efficiency e_r^{sw} is the radiation efficiency that accounts only for surface-wave loss, and not conductor or dielectric loss.)

   Repeat for a substrate relative permittivity of 10.8.

6) Repeat the previous problem, but now take the patch to be very small (to model an infinitesimal dipole), with $L = W = 0.01\lambda_0$. You are now comparing the exact surface-wave radiation efficiency of a dipole with the CAD formula for the surface-wave radiation efficiency of the dipole.

   Make plots for a substrate permittivity of 2.2 and 10.8.

Helpful Integration Tips

In Probs. 5 and 6 you will need to integrate a function in the complex plane along a straight line path. (The straight line path could be any one of the three line segments that make up the rectangular red path shown on slide 12 of Notes 24, or it could be the blue path shown on slide 12 of Notes 22.)

Consider a general integral along a straight line path in the complex plane from a complex point a to another complex point b, of the form

$$I = \int_a^b f(z)\,dz\,, \tag{1}$$

where f is a complex function of the complex variable z. (Note that the complex variable z here in this general notation will really be your complex variable k_t.)

We can use the following parameterization:

$$z = a + (b-a)t, \quad 0 \le t \le 1\,, \tag{2}$$

where t is a real variable that goes from zero to one. Note that

$$\frac{dz}{dt} = b - a\,. \tag{3}$$

We then use a change of variables from the complex z variable to the real t variable, by using

$$I = \int_0^1 f(z(t))\left(\frac{dz}{dt}\right)dt\,, \tag{4}$$

so that

$$I = (b-a)\int_0^1 F(t)\,dt\,, \tag{5}$$

where

$$F(t) \equiv f(z(t)). \tag{6}$$

The integral in Eq. (5) is along the real axis from $t=0$ to $t=1$. You can use any integration scheme that you wish to do this integration, including letting MATLAB do it for you.

Note that F is a complex function of the real variable t. If you wish, you can also write the integral as

$$I = (b-a)\int_0^1 \mathrm{Re}(F(t))\,dt + j(b-a)\int_0^1 \mathrm{Im}(F(t))\,dt\,. \tag{7}$$

In this form, we only need to numerically integrate two real-valued functions (one is $\mathrm{Re}(F(t))$ and the other is $\mathrm{Im}(F(t))$. This splitting of the integral in Eq. (5) into two parts would only be necessary if your numerical integration scheme cannot handle a complex function.