

**ECE 6345**  
**Spring 2024**  
**Class Project**

*Please check periodically to make sure that you have the latest version of the project (from the date at the top of this page). The project will be updated as corrections or changes are made.*

The purpose of this project is to design and analyze a  $4 \times 4$  corporate-fed microstrip antenna array as shown in Fig. 1 below, operating at 1.575 GHz. The design will be done using CAD formulas, and then the performance will be analyzed.

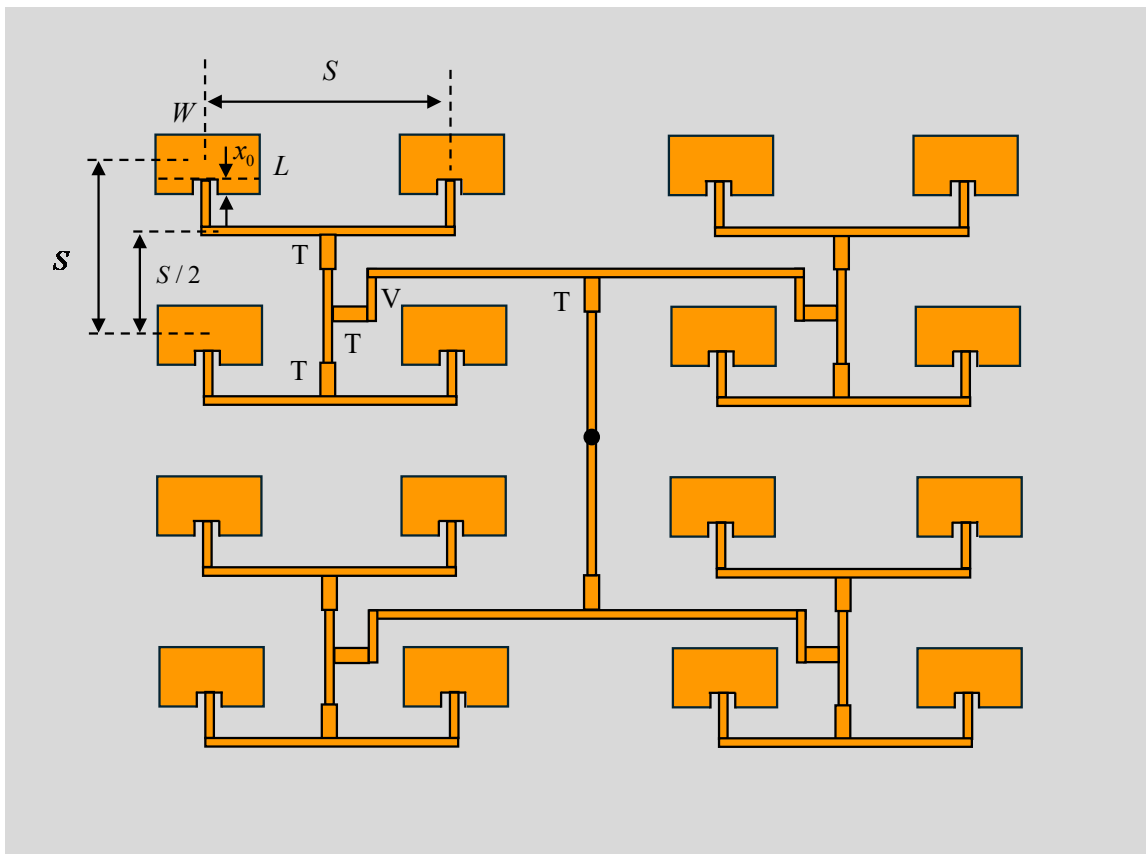


Fig. 1. Top view of  $4 \times 4$  corporate-fed microstrip antenna array.

## ARRAY DESIGN

The spacing  $S$  between the centers of the patches is  $\lambda_0$  in both the horizontal and vertical directions ( $\lambda_0$  is the free-space wavelength calculated at 1.575 GHz). Each antenna is a rectangular patch that is on a substrate of relative permittivity  $\epsilon_r = 2.2$  with a thickness of  $h = 0.1524$  cm (60 mils) and a loss tangent of 0.001. Assume that the conductivity of the patch and ground plane metal is  $3.0 \times 10^7$  S/m. The antenna array radiates with vertical polarization, with the short (vertical) dimension of each patch being the resonance dimension  $L$ . Each patch has  $W/L = 1.5$ . (This is the same patch as in the short course notes.) Each patch has a notch that allows for a feed at a position  $x_0$  (measured from the bottom edge) to give an input impedance of  $100 \Omega$  at resonance.

All microstrip lines (except for the transformer lines T) have a characteristic impedance of  $100 \Omega$ . The quarter-wave transformers (denoted as T) transform from a  $50 \Omega$  load impedance to a  $100 \Omega$  input impedance. At the center of the array is a  $50 \Omega$  feed point. The antenna array is designed to operate at 1.575 GHz, and each rectangular patch antenna is also designed to be resonant at  $f_0 = 1.575$  GHz. (This is the frequency where the patch cavities are resonant, so that the input resistance is maximum). Note that the patches are fed by microstrip lines, so ignore probe inductance here. The length of the vertical  $100 \Omega$  lines labeled as V is taken to be  $\lambda_0 / 4$ . All of the other line lengths for the  $100 \Omega$  lines can be calculated based on the given geometry information.

## PROJECT TASKS

### Task 1

- Design the dimensions and feed position of the patches ( $L, W, x_0$ ).
- Design the width of the  $100 \Omega$  lines.
- Design with width and length of the transformer lines.

### Task 2

- Plot the input impedance (real and imaginary part) of a single rectangular patch antenna as a function of frequency, from 1.50 to 1.65 GHz.
- Plot the SWR on the feeding microstrip line (which has  $Z_0 = 100 \Omega$ ) as a function of frequency, from 1.50 to 1.65 GHz. From this, calculate the bandwidth of the patch antenna (based on an SWR = 2.0 definition).
- Plot the E and H plane patterns of a single rectangular patch antenna at  $f_0$ . Plot in dB (normalized to zero dB at the peak) vs. angle  $\theta$  on a polar plot. Plot on a scale that has zero dB at the maximum and -40 dB at the center of the chart, with a scale of 10 dB per division.

### Task 3

- Plot the E and H plane patterns of the array at  $f_0$ . Plot in dB (normalized to zero dB at the peak) vs. angle  $\theta$  on a polar plot. Plot on a scale that has zero dB at the maximum and -40 dB at the center of the chart, with a scale of 10 dB per division.
- Calculate the gain of the array (the realized gain) at broadside at  $f_0$ .
- Plot the input impedance  $Z_{in}$  (real and imaginary part) of the array as a function of frequency, from 1.50 to 1.65 GHz.
- Plot the SWR on the feeding coax line (which has  $Z_0 = 50 \Omega$ ) as a function of frequency, from 1.50 to 1.65 GHz. From this, calculate the bandwidth of the array (based on an SWR = 2.0 definition).
- Plot the gain of the array (the realized gain) at broadside as a function of frequency, from 1.50 to 1.65 GHz.

### METHODS

The input impedance of each patch antenna can be calculated vs. frequency using the CAD circuit model for the patch. Transmission line theory can then be used to transform the patch impedances to the input feed point at the center of the array. In transmission line theory we have a standard formula for transforming a load impedance to the impedance at the input of the line. This formula can be used successively to get the input impedance at the central feed point. This formula is

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right). \quad (1)$$

Here  $\beta$  and  $Z_0$  are the phase constant and characteristic impedance on the line, which is either one of the main feed lines (the  $Z_0 = 100 \Omega$  lines) or the transformer lines (with  $Z_0 = 70.71 \Omega$ ). Note that for the main lines (M) and the transformer lines (T) we have

$$\beta_M = k_0 \sqrt{\epsilon_r^{\text{eff},M}}, \quad \beta_T = k_0 \sqrt{\epsilon_r^{\text{eff},T}}. \quad (2)$$

The effective relative permittivities can be found at  $f_0$  and then kept fixed (assume that they do not depend on frequency). The formula above for  $Z_{in}$  is for a lossless line, but this should be sufficiently accurate for the feed lines here.

Note that the lengths of all lines can be determined from the layout, starting with the known value of  $S$ , and the fact that all transformers are of length  $\lambda_{gT} / 4$  at the frequency  $f_0$ , where

$$\lambda_{gT} = \frac{\lambda_0}{\sqrt{\epsilon_r^{\text{eff},T}}}, \quad (3)$$

with the line lengths of the transformers being calculated by using the free-space wavelength and the effective relative permittivity on the transformer lines at  $f_0$ .

The far-field pattern of the array is the product of the element pattern  $E(\theta, \phi)$  (the pattern of a single patch) times the array factor for the array. The element pattern can be calculated by using the formula for the far-field of a rectangular patch (using, e.g., the electric current model discussed in Notes 8). The element pattern is then

$$E(\theta, \phi) = \sqrt{|E_\theta^p|^2 + |E_\phi^p|^2}, \quad (4)$$

where the ‘‘P’’ superscript denotes the far-field of the single rectangular patch (from Notes 8).

The array factor is

$$\text{AF}(\theta, \phi) = \sum_{n=1}^N A_n e^{jk_0(\sin \theta \cos \phi x_n + \sin \theta \sin \phi y_n)}, \quad (5)$$

where the sum is over all of the patches ( $N = 16$ ), and  $(x_n, y_n)$  are the coordinates for the center of patch  $n$  ( $n = 1, 2, \dots, 16$ ). (Note: It does not matter here how the patches are labeled (which one is called #1), or where the origin is. This will not change the magnitude of the array factor.)

The total far-field power density pattern (which is what gets converted to dB and then plotted) is then

$$P(\theta, \phi) = |E(\theta, \phi)|^2 |\text{AF}(\theta, \phi)|^2. \quad (6)$$

In dB, we have the normalized far-field power density pattern as

$$P_{\text{dB}}(\theta, \phi) = 10 \log_{10} \left( \frac{P(\theta, \phi)}{P(0, 0)} \right). \quad (7)$$

The directivity at broadside is defined as

$$D = D(0,0) = \frac{4\pi \left| \underline{E}^{\text{FF}}(0,0) \right|^2}{\int_0^{2\pi} \int_0^{\pi/2} \left| \underline{E}^{\text{FF}}(\theta,\phi) \right|^2 \sin\theta d\theta d\phi}, \quad (8)$$

where

$$\left| \underline{E}^{\text{FF}}(\theta,\phi) \right|^2 = \frac{1}{2\eta_0} \left( |E_\theta|^2 + |E_\phi|^2 \right). \quad (9)$$

The broadside gain  $G$  (realized gain, which accounts for the input impedance mismatch of the array) may be calculated at any frequency by using

$$G = G(0,0) = D(0,0) e_r \left( 1 - |\Gamma|^2 \right), \quad (10)$$

where  $e_r$  is the radiation efficiency of each patch (which may be calculated using the CAD formula for radiation efficiency at  $f_0$  and then assumed to be constant, and not a function of frequency) and  $\Gamma$  is the reflection coefficient looking into the array, assuming a feeding transmission line (coax) with a characteristic impedance of  $Z_0^{\text{F}} = 50 \Omega$ . Hence, at any frequency we have

$$\Gamma = \frac{Z_{\text{in}} - Z_0^{\text{F}}}{Z_{\text{in}} + Z_0^{\text{F}}}, \quad (11)$$

where  $Z_{\text{in}}$  will be a function of frequency.

The directivity at broadside  $D(0,0)$  may be assumed to be constant, calculated at  $f_0$ . However, the reflection coefficient  $\Gamma$  is a definite function of frequency, and this will be responsible for the gain loss of the array as the frequency is varied away from  $f_0$ .

Please use TXLINE for the design of all the microstrip lines. Assume that the lines have the same conductivity as the patch ( $3.0 \times 10^7 \text{ S/m}$ ), and remember that the substrate has a loss tangent of 0.001. Also, assume that the thickness of the copper lines in TXLINE is  $17.5 \mu\text{m}$  (17.5 microns).

## FORMAT GUIDELINES

It is recommended that you use the same type of format to write your project as is used for this project description. You can use this project description as your template. It is recommended that you use MathType for all equations.