# Name: \_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Dec. 3, 2014

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality must have the Internet access disabled.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown (this includes showing all relevant paths in the complex plane that you use to solve a problem).
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

Problem 1 (30 pts.)

Evaluate the following integrals.

a) 

b) 

c)  (This integral is defined in the Cauchy principal value sense.)

**Room for Work**

**Part (a)**

Close the contour in the UHP. We then have



We have

.

Hence, we have

.

**Part (b)**

Close the contour in the UHP (arbitrarily). We then have



We have





This gives us

.

We then have

.

The final result is then



or

.

**Part (c)**

Close the contour in the UHP, deforming counterclockwise below a small semicircle at the origin. We then have

.

We also have

.

Hence, we have

.

Problem 2 (40 pts.)

Consider the following function:

.

a) Find the Laurent series expansion about the point *z* = 0, valid in the region 0 < *z <*1.

b) Find the Laurent series expansion about the point *z* = 0, valid in the region *z >* 1.

c) Find the first two terms of the Taylor series expansion about the point *z =* 3.

d) For the Taylor series in part (c), what is the radius of convergence of the Taylor series?

**Room for Work**

**Part (a)**



so



**Part (b)**



and hence

,

so that

.

**Part (c)**

.

Hence, we have

.

**Part (d)**

The radius of convergence is the distance from the point *z* = 3 to the nearest singularity, at *z* = 0. Hence we have

.

Problem 3 (30 pts.)

Consider the electrostatic problem shown below. The semicircular arc (of radius 1) is a perfect insulating boundary (perfect magnetic conductor). The bottom of the structure (the part on the *x* axis) is at a potential of either 1V (*x* > 0) or -1V (*x* < 0). (Note: There is a small insulating gap between the two plates on the *x* axis, to prevent them from shorting together.)

Solve for the potential at any point (*x*,*y*) inside the structure.

Hint: Consider the mapping

.

PMC

*x*

*y*

-1V

+1V

1

**Room for Work**

The mapping transforms the original problem into the following problem in the *w* plane:

PMC

*u*

*v*

-1

+1

*v* = π

In the *w* plane we have

.

Hence, we have

.

This can also be written as

.

Note that for the inverse tangent we use

.