# Name: \_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 8, 2017

1. This exam is open-book and open-notes. Calculators are allowed. Cell phones are not allowed. Laptops are allowed but they must not be used to access the Internet during the exam. They can only be used to display material such as the class notes that you already have stored on your laptop.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

Problem 1 (25 pts.)

Evaluate the following Cauchy principal value integral by using complex analysis:

.

Note that there are poles on the real axis, so the integral is defined in the Cauchy principal value sense.

Solution

We choose a contour that detours above the poles on the real axis at *x* = -2 and *x* = 1, and then is closed in the upper half plane, enclosing the pole at *z* = *i*. We then have



so that

.

 A residue calculation yields



****

****.

We then have



or

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or



or

.

This simplifies to

.

Problem 2 (25 pts.)

Evaluate the following integral using complex analysis:

.

Solution

We choose to put the branch cut along the positive real axis, and assume that the angle θ is in the range 0 < *θ* < 2*π*. We then choose a contour that runs along the real axis from the origin to infinity just above the branch cut, then arcs along a large circular arc to reach the positive real axis at infinity just below the real axis, and then comes in along the real axis just below the branch cut to reach the origin. The origin is surrounded by an arc of a circle of vanishingly small radius. The small arc patch gives a zero contribution as the radius of the small circle approaches zero. This path encloses a simple pole at *z* = *i* and a simple pole at *z* = -*i*. We then have



so



or

.

We then have



.

Hence,

.

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Simplifying, we have



or



or



or

.

Problem 3 (25 pts.)

Consider the geometry shown below. It consists of two semi-infinite metal plates, one for *x* > 0 and one for *x* < 0. The geometry is infinite in the *z* direction. Note that there is an insulating gap at the origin.

a) Use conformal mapping to solve for the potential in the region *y* > 0. As part of your solution, draw carefully what the geometry looks like in the *w* plane.

b) Solve for the surface charge density on the upper surface of the metal plate for *x* > 0.

Hint: Consider the mapping *w* = ln (*z*).

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Solution

The mapping maps the geometry into an infinite parallel-plate structure, with 1 [V] along a conductor in the *w* plane that lies on the *v* = 0 axis, and a conductor of 0 [V] that lies along the horizontal line *v* = *π*. The potential is then

.

Hence, in the original *z* plane we have

.

Hence, we have



or

.

For the surface charge density we have

.

Hence we have



so

.

Problem 4 (25 pts.)

Consider the function

.

Find the complete Laurent series expansion for the functionthat is valid in the region.

Solution

We can write

.

Hence, we need to find a Laurent series expansion for the term in parenthesis. We use an partial fraction expansion to obtain

.

We then write



or

.

Using a geometric series, we then have

.

Hence, we have



or



or



or



or

.

**Room for Work**