# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 11, 2019

1. This exam is open-book and open-notes. Calculators are allowed. Laptops and cell phones (or any devices that have communication functionality) are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Perform all your work on the exam in the space allowed.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

Find the residues of the following functions at the origin:

(a) 

(b) 

(c) 

**Part (a)**

There is a simple pole at . Hence we can use



so

.

**Part (b)**

There is a double pole at . Hence we can use



so

.

**Part (c)**

There is an essential singularity at . Hence we cannot use the standard formula that holds for a pole of finite order *m*. We can try expanding the function out:

.

We need to identify the  coefficient.

We have

.

Hence,

.

**Problem 2 (25 pts.)**

Consider the following function:

.

a) Derive the first three terms of the Taylor series in the region , expanding about the point .

b) Derive the first three terms of the Laurent series in the region , expanding about the point .

c) Derive the first three terms of the Laurent series in the region , expanding about the point .

**Part (a)**

We can expand the function as

.

The first three terms are

.

**Part (b)**

We can expand the function as

.

The first three terms are

.

**Part (c)**

We can expand the function as

.

Hence, the Laurent series only has three terms. The Laurent series is

.

**Problem 3 (25 pts.)**

Evaluate the following integrals:

a) 

b) 

c) 

**Part (a)**

We can write

.

We can close in the upper half plane with a large semicircle, and go counterclockwise around the contour. We then have



We have



Hence we have



or

.

**Part (b)**

We can write

.

We then close in the upper half plane, and use a small semicircle to detour around the pole on the real axis at . A counterclockwise integration gives us



so

.

Hence

.

We have

.

Hence, we have

.

**Part (c)**

We can write

.

We can choose a branch cut on the positive real axis, and then integrate along the top and bottom of it, and close with a large circle at infinity, integrating counterclockwise around the entire path. We then have

.

We also have



.

Hence we have



This gives us



This may be written as



or



or



or



or

.

Problem 4 (25 pts.)

Consider the semi-infinite parallel-plate structure shown below (the same structure that was considered in the class notes).

Drive an expression for the magnitude of the electric field along the center of the structure (the horizontal centerline, at *y* = 0), as a function of *x*. That is, drive an expression for .

You may leave your answer in terms of the variable *u*, as long as you clearly relate *u* to *x* for points along the centerline.

(Hint: consider what happens when you set *v* = 0 in the mapping.)



**Solution**

We can use

.

The mapping is

.

Hence,

,

and thus

.

We then have

 

.
We also have

.

Hence, we have

.

Along the centerline , we are on the centerline . Hence, we have

,

where

.