# Name: \_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 10, 2021

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are in “airplane” mode and are not used to communicate in any way with anyone. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

Consider the following function:

.

The branch cut for the Ln function is chosen to be vertically downward from the branch point at .

a) Derive the first three terms of the Laurent series expansion of this function about the point .

b) Where will this Laurent series converge?

c) What type of singularity is at ?

d) Find the residue at the point .

**Solution**

**Part (a)**



Collecting terms, we have



or



**Part (b)**

The Laurent series will converge for

.

**Part (c)**

The singularity at  is a pole of order 2 (double pole).

**Part (d)**

The residue is -1/2.

**Problem 2 (25 pts.)**

Evaluate the following integral:

.

**Solution**

The integrand has a removable singularity at the origin and simple poles at .



We can then write

,

where  is the integral. Now there is a simple pole at the origin.

We then deform around the three poles, choosing (arbitrarily) to detour below them. We also close the contour with a large semicircle in the upper half plane.

We then have



or



For the residues we have:







Hence,

.

We then have

.

Problem 3 (25 pts.)

Consider the following function:

.

For a point *A* on the positive real axis with , both the ln function and the square root function have positive real values.

Evaluate the function  at the point , assuming that we arrive at this point by starting at a point *A* on the positive real axis and then moving smoothly on the Riemann surface to arrive at the point *z*, by using the paths indicated below. (There are four parts to this problem, each with a different path.)

**Part (d)**

**Part (c)**

**Part (a)**

**Part (b)**

**Solution**

,

where  and  are the magnitude and argument of .

Hence,

 .

At the point  we have  and .

Hence, we have

.

**Part (a)**





**Part (b)**





**Part (c)**





**Part (d)**





Problem 4 (25 pts.)

Consider the problem shown below, consisting of a “twin lead” transmission line. The mapping



maps the twin lead into the coaxial geometry as shown. Note that the right conductor in the *z* plane gets mapped into the inner conductor in the *w* plane.

Assume that the left conductor of the twin lead is at a potential of 1 volts and the right conductor is at a potential of zero volts.

Solve the potential at any point *z* in the twin lead problem. You may leave your answer in terms of *a* and .













**Solution**

In the *w* plane, the potential is

.

Enforcing the boundary conditions, we have



.

The solution is:



.

Hence, in the *w* plane we have

.

In the *z* plane we then have

.

We have



so that

.

Hence, we have



with

.

**Room for Work**