# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 9, 2016

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown (this includes showing all relevant paths in the complex plane that you use to solve a problem).
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

Problem 1 (25 pts.)

Evaluate the following integral as Ω → ∞.

.

The path *C* is shown below, which lies along a 45o line and goes through the origin, going from -(1+*i*)∞ to (1+*i*)∞.

As part of your solution, clearly identify and sketch the path of steepest descent and the path of steepest ascent.



**Solution**

We have





.

The saddle point is located at

.

We then have



.

There is also a simple pole located at *z* = ± *i*.

The SDP comes from

.

The may be simplified to



or

.

We have:

SDP: 

SAP: .

The saddle-point recipe is

.

We then have

.

The total integral is given by

.

The residue is

.

Hence we have

.

Problem 2 (25 pts.)

Asymptotically evaluate the following integral as Ω → ∞.

.

The first two terms of the asymptotic expansion are sufficient.

**Solution**

Use  to obtain

.

Hence we have



where



Therefore,

.

where



.

We have

.

Hence, we have

.

The first two terms are

.

Problem 3 (25 pts.)

Consider the Frobenius series solution about the point *x* = 0 of the following differential equation:

.

a) Classify the point *x* = 0 as an ordinary point, a regular singular point, or an irregular singular point.

b) Classify the differential equation as case 1, case 2, or case 3.

c) Find the Frobenius series solution for *y*1 (*x*), i.e., the solution that has the largest value of *α*. Find the first three terms of the series solution. Take *a*0 = 1.

**Solution**

The point *x* = 0 is a regular singular point.

In the Frobenius method we start by assuming

.

The Frobenius method gives us the result

.

The indicial equation is thus

.

The solution for the exponents terms is

.

Therefore, this is case 2.

The recurrence formula is

.

The solution is

.

Hence, with *a*0 = 1 we have

.

The first few terms are

.

Problem 4 (25 pts.)

Consider an infinite transmission line that runs from *z* = -∞ to *z* = ∞.

If there is a distributed parallel current source *Isd*(*z*) on the line, then the voltage on the line satisfies the differential equation

.

A Green’s function is defined that gives the voltage *V*(*z*,*z*′) at *z* due to a 1A parallel current source at *z*′ (please see the first figure below). The Green’s function thus corresponds to a distributed parallel current source of the form

.

a) Solve for the Green’s function , for both *z* > *z*′ and *z* < *z*′.

b) Assume that a distributed parallel current source  exists on the line (please see the second figure below). The current source has the form (where *A*1 is a constant)

.

Solve for the voltage *V*(*z*) on the transmission line in the region *z* > *l*/2.





**Solution**

The two solutions in method 1 are:



.

The Wronskian is

.

Also, we have that

.

Hence, the Green’s function is



This can also be written as



where

.

The voltage for *z* > *l* is



so that



Hence, after evaluating the integral, we have

.

This may be written as

.

**Room for Work**