# Name:\_\_\_\_\_\_\_\_\_\_\_Solution\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 11, 2019

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown (this includes showing all relevant paths in the complex plane that you use to solve a problem).
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (20 pts.)**

Consider the following integral as *x* → ∞.

.

Derive the first three terms of the asymptotic series as *x* → ∞.

**Solution**

Use integration by parts, starting with  and .

The result is

.

**Problem 2 (20 pts.)**

Consider the following integral as Ω → ∞.

.

Derive the first three terms of the asymptotic series as Ω → ∞.

**Solution**

Use Watson’s Lemma with . That is, we use



so that

.

We have

.

Watson’s lemma then gives us

.

**Problem 3 (20 pts.)**

Evaluate the leading term of the asymptotic expansion of the following integral as Ω → ∞ using the method of steepest descent:

.

The path *C* is along the real axis from 0 to *π*.

As part of your solution, clearly identify and sketch the path of steepest descent and the path of steepest ascent. Explain how you are determining the departure angle *θ*SDP.



**Solution**

We have:



The SDP and SAP are determined by

.

We also have

.

Fro this we see that the SDP is the real axis and the SAP is the vertical line . We thus conclude that .

We then have

.

**Problem 4 (20 pts.)**

Consider the following differential equation:

,

with the boundary conditions



Find the Green’s function  for this problem by using “method 1”.

**Solution**

We have, for the solutions of the homogenous differential equation:



The Wronskian is

.

We then have, using  and :



**Problem 5 (20 pts)**

Consider an infinite transmission line that runs from *z* = 0 to *z* = ∞. The transmission line is fed by a parallel (shunt) 1A current source at *z* = *z*′, as shown in the figure below. On the left end of the line at *z* = 0 there is a short circuit. On the right end, the line runs to infinity, so there can only be a wave traveling in the positive *z* direction to the right of the source.

Solve for the voltage *V* (*z*) on the transmission line (for both *z* < *z*′ and *z* > *z*′) using “method 1”.



**Solution**

We have, for the solutions of the homogenous differential equation:



The Wronskian is

.

We then have, using  and :



**Bonus Problem (20 pts)**

A tube is being used to carry a current in the *z* direction at some frequency *ω*. The tube is infinite in the *z* direction, and there is no variation in the *z* or *φ* directions. The tube consist of a copper conductor of inner radius *a* and outer radius *b* and conductivity *σ* as shown below. There is an electric field *Ez* inside the conductor. The current *Jz* inside the conductor obeys Ohm’s law. The tube is excited by a source that is inside of the tube. Because of this, we can assume that at the outer boundary of the tube (*ρ* = *b*) the electric field *Ez* satisfies the condition that the derivative with respect to the radial distance *ρ* is zero.

Write down an expression for the electric field inside the tube in terms of the Bessel functions. Normalize your solution so that the electric field is 1 [V/m] at the inner boundary (*ρ* = *a*).



**Solution**

We have

.

Because of the boundary condition at the outer surface, we have

.

Hence we have

.

**Room for Work**