# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Dec. 11, 2020

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are not used to communicate in any way with anyone other than the instructor. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.
6. At the end of the exam you will be asked to scan your pages and convert them to a single pdf file, and then email this file to the instructor (djackson@uh.edu).

**Problem 1 (25 pts.)**

a) Find the first two terms of the asymptotic series for the following integral (where *A* is a fixed constant) as Ω gets large:

.

b) Assume we keep only the first term of the asymptotic series. Derive a formula that asymptotically predicts what the error is in using this leading term to estimate the integral, as Ω gets large.

**Solution**

**Part (a)**

Use integration by parts:



Hence,



**Part (b)**

The error estimate is the next term neglected. Hence we have



**Problem 2 (25 pts.)**

Find the first two terms of the asymptotic series for the following integral, as Ω gets large:

.

**Solution**

Use Watson’s lemma:

,

where .

Let :

 .

We have



Watson’s lemma states that



provided that

.

Hence, we have



This gives us

.

Problem 3 (25 pts.)

Consider the following integral:

.

a) Sketch the steepest-descent path for this problem and identify the saddle point.

b) Asymptotically evaluate this integral (i.e., find the leading term of the asymptotic expansion) using the method of steepest descent.

**Solution**

We have:







.

The saddle point is at

.

(There are saddle point located at  for any integer n, but this is the only saddle point that appears along the given path.)

We also have

.

Hence, we have



or

.

The SDP (and SAP) are given by



or



We can see which one is the SDP by using

.



We then can see that

.

The SDP recipe is

.

We then have

.

Problem 4 (25 pts.)

Consider the following differential equation:

,

where *p* is a positive real number and . The function *u* is assumed to satisfy a boundary condition at , which is  as .

a) Solve for the Green’s function , using “method 1”.

b) Assume that we now have  (with  being zero for *x* outside this range). Solve for  in the region .

**Solution**

**Part (a)**

From method 1 we have:



where

.

In our case we have





.

We also have (keeping in mind the boundary condition at )



.

The Wronskian is

.

We then have



**Part (b)**

,

and thus we have

.

This gives us

.

We then have



or

.