# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 13, 2021

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are not used to communicate in any way with anyone other than the instructor. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

a) Find the first two nonzero terms of the asymptotic series for the following integral as Ω gets large:

.

b) Assume we keep only the first term of the asymptotic series. Give a formula that asymptotically predicts what the error is in using this leading term to estimate the integral, as Ω gets large.

**Solution**

**Part (a)**

In order to use Watson’s Lemma, use the substitution

 

.

Also use

.

This gives us

.

Note that we can extend the upper limit without affecting the asymptotic expansion, so that



We have, using a Taylor series,

.

Hence, from Watson’s Lemma, we have

.

**Part (b)**

If we keep only the first (leading) term, of the asymptotic series, the error estimate is then given by the second term. Hence, we have

.

**Problem 2 (25 pts.)**

Find the leading term of the asymptotic series for the following integral, as Ω gets large:

.

The original path is along the imaginary axis from  to .

As part of your solution, show what the SDP and SAP paths look like.

**Solution**

In the method of steepest-descent, we have



.

We have



.

We see that there is a saddle point at

.

We also have

.

We see that

.

Hence, we know that

.

The SDP and SAP paths come from .

To find the *u* and *v* functions, we use

.

This gives us



.

Hence, the SDP and SAP correspond to



or



so that



or



or

.

Looking at the *u* function, we see that



We thus conclude that

 .

We then use the steepest-descent recipe:

.

This gives us



or

.

Problem 3 (25 pts.)

Consider the following differential equation:

.

The boundary conditions are:



Find the Green’s function  for this problem using “method 1”.

**Solution**

For the homogeneous solution, we have



so



or



so

.

Hence, we can choose



.

The Wronskian is

.

The Green’s function recipe is



For our differential equation, we compare with

.

We then see that



 .

Hence, we have



Simplifying, we have



Problem 4 (25 pts.)

Consider the following differential equation:

,



a) Solve for the Green’s function , using “method 2” (an eigenfunction expansion).

b) Assume that we now have  (with  being zero for *x* outside this range). Solve for  in the region .

**Solution**

**Part (a)**

The eigenvalue problem is

.

We define



where

.

We have for the eigenvalues and eigenfunctions



.

Hence,



so

.

The recipe is

.

In our case, we have

.

We also have



where the Kronecker delta symbol is defined as



Hence, we have

 .

**Part (b)**

We have

.

Integrating the Green’s function, we have

.

Hence, we have

.