# ECE 6382

# Fall 2023

## Homework Set #4

Homework problems are from *Mathematical Methods for Physicists*, 7th Ed., by Arfken, Weber, and Harris.

**Chapter 11, Section 6, Singularities**

 **Prob. 11.6.5**

Note: Please include the point at infinity in your considerations.

**Chapter 11, Section 7, Residue Theorem**

**Prob. 11.7.2**

**Prob. R1**

Evaluate the following integral:

,

where the contour *C* runs clockwise around the origin.

**Numerical Calculation of Residues**

**Prob. N1**

Consider the function

.

This function has a simple pole at the origin, with residue 1.

Make a table showing the value of the residue that you predict numerically for the pole at the origin, by sampling at *N* symmetric locations that are located on a circle of radius *r*. (Please see the formula in Notes 10.) Choose the first point to be located on the *x* axis (an angle of *θ* = 0) so that (in the notation of the formula) Δ*z* = *r*.

In the first column of the table, choose *r* = 0.1, and let *N* = 1, 2, 4, 8.

In the second column of the table, choose *r* = 0.01, and let *N* = 1, 2, 4, 8.

In the third column of the table, choose *r* = 0.001, and let *N* = 1, 2, 4, 8.

In the fourth column of the table, choose *r* = 0.0001, and let *N* = 1, 2, 4, 8.

Keep at least 10 significant figures in your results.

**Prob. N2**

Consider the function

.

This function is analytic everywhere (and hence on the real axis) and periodic on the real axis with a period of *π*. Therefore, according to the discussion in Notes 10, using the midpoint rule of integration should work unusually well if we integrate over a complete period. Verify this by calculating the integral of this function on the real axis from zero to *π* using the midpoint rule with *N* = 1, 2, 4, 8 intervals. Make a table showing the result for each *N*, along with the percent error in the result.

Then make the same type of table using a numerical integration of the same function with the midpoint rule, integrating from zero to 0.9*π*. (The function is not periodic over this interval.)

Keep at least eight significant figures in your results.

Note that the exact answers can be found by using

.

**Chapter 11, Section 8, Evaluation of Definite Integrals**

**Prob. 11.8.10**

**Prob. 11.8.14** (You may assume that *p* > 0.)

**Prob. 11.8.15**

**Prob. 11.8.20**

**Prob. 11.8.22** (Note that *n* is an integer that is greater than or equal to 2.)

**Chapter 11, Section 7, Mittag-Leffler Theorem**

**Prob. M1**

Find the Mittag-Leffler expansion of the following function, and show that it is the same as what you would get from using a partial fraction expansion.

