ECE 6382

Fall 2023

Homework Set #4

Homework problems are from *Mathematical Methods for Physicists*, 7th Ed., by Arfken, Weber, and Harris.

Chapter 11, Section 6, Singularities

Prob. 11.6.5

Note: Please include the point at infinity in your considerations.

Chapter 11, Section 7, Residue Theorem

Prob. 11.7.2

Prob. R1

Evaluate the following integral:

$$I = \int_C e^{1/z} dz \,,$$

where the contour C runs clockwise around the origin.

Numerical Calculation of Residues

Prob. N1

Consider the function

$$f(z) = \frac{1}{z^2 + \sin z} \,.$$

This function has a simple pole at the origin, with residue 1.

Make a table showing the value of the residue that you predict numerically for the pole at the origin, by sampling at N symmetric locations that are located on a circle of radius r. (Please see the formula in Notes 10.) Choose the first point to be located on the x axis (an angle of $\theta = 0$) so that (in the notation of the formula) $\Delta z = r$.

In the first column of the table, choose $r = 0.1$, and let $N = 1, 2, 4, 8$.
In the second column of the table, choose $r = 0.01$, and let $N = 1, 2, 4, 8$.
In the third column of the table, choose $r = 0.001$, and let $N = 1, 2, 4, 8$.
In the fourth column of the table, choose $r = 0.0001$, and let $N = 1, 2, 4, 8$.

Keep at least 10 significant figures in your results.

Prob. N2

Consider the function

$$f(z) = \sin^2 z\,.$$

This function is analytic everywhere (and hence on the real axis) and periodic on the real axis with a period of π. Therefore, according to the discussion in Notes 10, using the midpoint rule of integration should work unusually well if we integrate over a complete period. Verify this by calculating the integral of this function on the real axis from zero to π using the midpoint rule with $N = 1, 2, 4, 8$ intervals. Make a table showing the result for each N, along with the percent error in the result.

Then make the same type of table using a numerical integration of the same function with the midpoint rule, integrating from zero to 0.9π. (The function is not periodic over this interval.)

Keep at least eight significant figures in your results.

Note that the exact answers can be found by using

$$\int_0^b \sin^2 x\,dx = \frac{b}{2}\left(1 - \frac{\sin(2b)}{2b}\right).$$

Chapter 11, Section 8, Evaluation of Definite Integrals

Prob. 11.8.10
Prob. 11.8.14 (You may assume that $p > 0$.)
Prob. 11.8.15
Prob. 11.8.20
Prob. 11.8.22 (Note that n is an integer that is greater than or equal to 2.)

Chapter 11, Section 7, Mittag-Leffler Theorem

Prob. M1

Find the Mittag-Leffler expansion of the following function, and show that it is the same as what you would get from using a partial fraction expansion.

$$f(z) = \frac{z}{z-1} + \frac{z^2+1}{z^2+4}$$